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ROBUSTNESS OF MODEL REFERENCE ADAPTIVE SCHEMES
WITH RESPECT TO MODELING ERRORS

BY

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THESIS

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Thesis Adviser: Professor P. V. Kokotovic

Urbana, Illinois

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NOTATION

Vectors and scalars are generally represented by lower case letters and matrices by upper case letters. The derivative with respect to time of a vector x is denoted by $\dot{x}$. The transpose of a matrix A is denoted by A^T. The absolute value of a scalar f is denoted $|f|$. Norm of a matrix or vector M is denoted $\|M\|$. Notation for a set $\mathcal{D}$ composed of all objects u having some property P is

$$\mathcal{D} = \{u : u \text{ has property P}\}.$$

A function $f(\mu)$ is $O(\mu^k)$ if there exist positive constants μ^* and c such that the norm $\|f\|$ satisfies

$$\|f\| \leq c\mu^k \quad \text{when} \quad k > 0$$

or

$$\|f\| \geq c\mu^k \quad \text{when} \quad k < 0$$

for all $\mu \in (0,\mu^*]$. The expression $Q > 0$ means that Q is a positive definite matrix. The i-th eigenvalue of a matrix R is denoted as $\lambda_i(R)$.

1. INTRODUCTION

1.1. Problem Description

In the last twenty-five years adaptive systems theory has received a lot of attention by theoreticians and practitioners. Global stability of MRAS (Model Reference Adaptive Schemes), an open problem for almost two decades, was recently solved for both continuous and discrete SISO (Single-Input Single-Output) systems [1-16]. Moreover, different adaptive schemes developed independently have been shown to be equivalent and diverse approaches have been unified [13], [17-19]. However, there still remains a significant gap between the available theoretical methodologies and potential applications of such adaptive schemes. Global stability properties are guaranteed under the "matching assumption" that the model order is not lower than the order of the unknown plant. Since this restrictive assumption is likely to be violated in applications, it is important to determine the robustness and performance of adaptive schemes with respect to such modeling errors.

Recently several attempts have been made to formulate and analyze reduced-order MRAS [20-24]. The results of such studies depend on the characterization of the model-plant mismatch. This thesis is divided into six sections. Different characterizations are given for the model-plant mismatch in Sections 2 to 5 and 6. ~~Our~~ The characterization in Sections 2 to 5 assumes a separation of time scales between the modeled and unmodeled phenomena. The order of the model is equal to the order of the slow part of the unknown plant and the model-plant mismatch is due to the fast part of the plant. In most applications the slow part consists of "dominant" modes, while the neglected fast modes are considered as "parasitics." ~~Our~~ The approach is

asymptotic. A crucial parameter in our analysis is the "speed ratio" μ of the slow versus the fast phenomena. The limit as $\mu \to 0$ means that the fast part of the plant reaches its steady-state instantaneously, that is, the plant reduces to its slow part. The fictitious "reduced-order" plant is thus obtained when in the actual plant $\mu > 0$ is replaced by $\mu = 0$. This singular perturbation approach is a convenient parameterization of the model-plant mismatch because it allows an asymptotic analysis using the limit as $\mu \to 0$. In our formulation adaptive schemes are designed for the reduced order plants and then applied to the actual plants. They are considered robust if the error in their performance, due to model-plant mismatch, is $O(\mu)$. In Section 6 the model-plant mismatch is characterized by neglected interconnections between different subsystems. Adaptive schemes are first designed for the decoupled subsystems and then applied to the overall system with the interconnections.

1.2. Section Review

In Section 2 we analyze the behavior of continuous-time identifiers and adaptive observers with respect to modeling errors, consisting of fast parasitics which are weakly observable in the plant output. The adaptive schemes considered are shown to be robust provided the input signal is dominantly rich, that is it is rich for the dominant modes, but does not contain high frequencies in the parasitic range. The bounds indicate possibilities for reducing the error by a proper choice of the input signal.

The performance of reduced-order discrete-time parallel and series parallel adaptive identifiers with respect to weakly observable parasitics is

treated in Section 3. While in a deterministic environment with no modeling error the two schemes give identical results, their performance is different when used in a deterministic environment with modeling error. Detailed bounds on the composite output/parameter error are established. The dependence of these bounds on the initial identification error, the characteristics of the input, and the speed ratio μ are shown to be crucial.

In Section 4, it is shown that the weak observability assumption used in Sections 2 and 3 is crucial. That is when the parasitics are strongly observable the adaptive schemes are no longer robust. A redesign procedure has been introduced which re-establishes the robustness of identifiers and adaptive observers with respect to fast strongly observable parasitics.

The effects of unmodeled fast dynamics on the stability and performance of adaptive control schemes are analyzed in Section 5. In the regulation problem global stability properties are no longer guaranteed, but a region of attraction exists for exact adaptive regulation. In the case of tracking a more robust adaptive law is proposed which guarantees the existence of a region of attraction from which all signals converge to a residual set which contains the equilibrium for exact tracking. The size of this set depends on design parameters, the frequency range of parasitics, and the reference input signal characteristics.

In Section 6 the problem of regulation and tracking of a class of large-scale linear time invariant systems with unknown parameters is considered. An approach is developed for adaptive regulation and tracking using decentralized adaptive controllers. Sufficient conditions are established in a form of algebraic criteria which can guarantee stability or boundedness under certain structural perturbations.

Finally, Section 7 contains the conclusions and possible directions for future research.

2. CONTINUOUS-TIME IDENTIFIERS AND ADAPTIVE OBSERVERS

2.1. Characterization of the Mismatch

Suppose that an (n+m)th order linear time invariant plant has n slow ("dominant") and m fast ("parasitic") modes, that is n of its eigenvalues are O(1) and the remaining m are $O(\frac{1}{\mu})$, where μ is a small positive scalar. Without loss of generality such a plant can be represented by the standard singular perturbation form

$$\dot{x} = A_{11}x + A_{12}x_f + B_1u \tag{2.1}$$

$$\mu\dot{x}_f = A_{21}x + A_{22}x_f + B_2u \tag{2.2}$$

where x, x_f are n and m vectors, respectively, and u is an r input vector. In this form the dominant and the fast parts do not appear explicitly, but state variables x, x_f and parameter μ have clear physical meaning. Typically μ represents small time constants, masses, inertias etc. [25]. It is known from [25] that the contribution of the fast modes to x is only $O(\mu)$ and hence x can be used as the state of the dominant part of the plant. State x_f is formed of a "fast transient" and a "quasi-steady state" defined as the solution of (2.2) with $\mu\dot{x}_f = 0$. This motivates the definition of the fast parasitic state as

$$\eta = x_f + Lx + A_f^{-1}B_fu \tag{2.3}$$

where L is required to satisfy [26]

$$A_{22}L - A_{21} + \mu LA_{12}L - \mu LA_{11} = 0. \tag{2.4}$$

From (2.4) and assuming that A_{22}^{-1} exists we see that

$$L = A_{22}^{-1}A_{21} + O(\mu) \tag{2.5}$$

and hence, for μ small, η is the difference between x_f and its "quasi-steady state" $-A_{22}^{-1}A_{21}x - A_{22}^{-1}B_2u$. Defining

$$A = A_{11} - A_{12}L, \quad B = B_1 - A_{12}A_f^{-1}B_f, \quad A_f = A_{22} + \mu LA_{12},$$
$$B_f = B_2 + \mu LB_1, \quad H = A_{12} \tag{2.6}$$

and substituting (2.3) into (2.1), (2.2) we obtain a representation of (2.1), (2.2) with the dominant and parasitic parts appearing explicitly

$$\dot{x} = Ax + Bu + H\eta \tag{2.7}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1}B_f\dot{u} \tag{2.8}$$

as shown in Fig. 2.1. The parasitic state η enters the dominant part as a disturbance input $H\eta$. The parasitic part of the plant is driven by the input $\mu\dot{u}$. The steady-state gain in the disturbance path is

$$-HA_f^{-2}B_f \tag{2.9}$$

and will be shown to play a crucial role in the identification/observation error due to parasitics. From (2.7), (2.8) and Fig. 2.1 it can also be anticipated that another crucial factor will be the fequency content of u(t). If u(t) contains frequencies as high as $O(\frac{1}{\mu})$, then $\mu\dot{u}$ is O(1) and the steady state of η is O(1), that is the influence of the parasitics is significant.

We can now formulate the model-plant mismatch problem as follows. The goal is to identify and observe only the nth order dominant part of the plant. Hence the order of the adjustable model is chosen to be n. An adaptive scheme is then designed ignoring the parasitics, that is assuming $\mu = 0$. Our problem is to analyze the performance of this scheme when applied to the plant with parasitics, that is with $\mu > 0$.

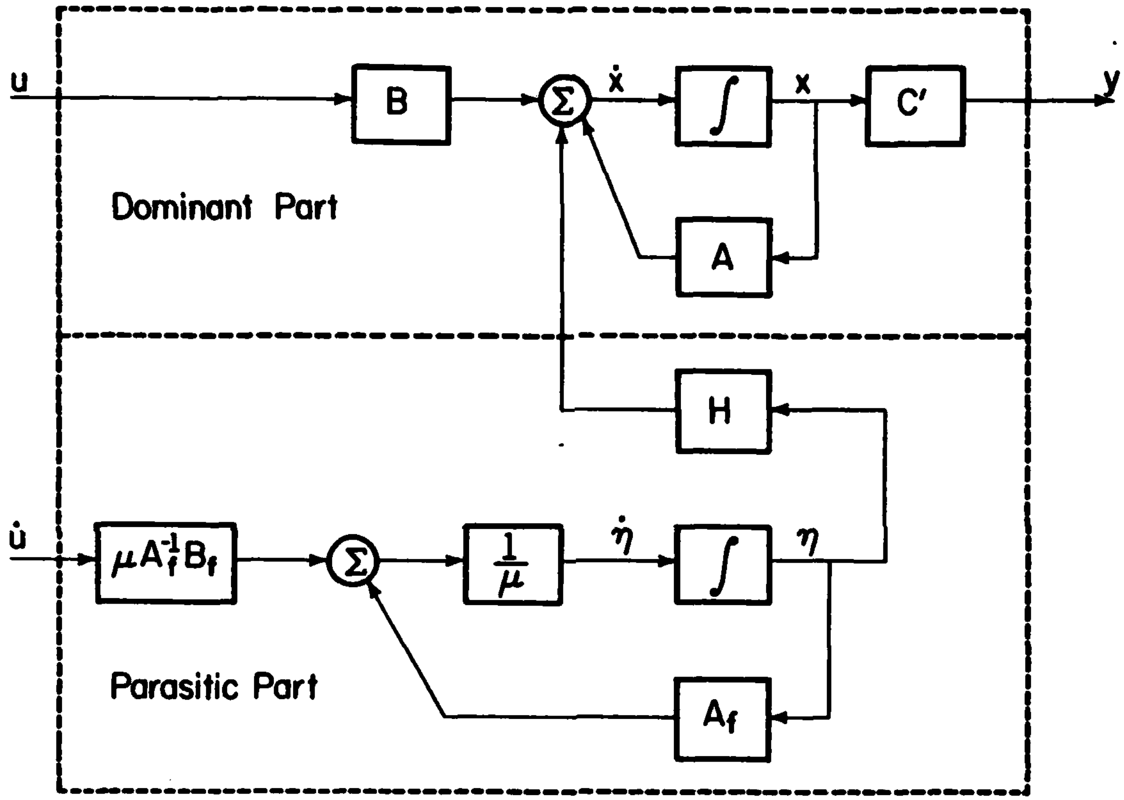


Fig. 2.1. Representation of the plant based on (2.7), (2.8), and (2.49).

We address this problem under the following assumptions:

Assumption I: The plant with parasitics is asymptotically stable, that is

$$\mathrm{Re}\lambda(A) < 0, \qquad \mathrm{Re}\lambda(A_f) < 0. \tag{2.10}$$

Assumption II: The pair (A,B) of the dominant part is controllable.

Noting from (2.5),(2.6) that A, A_f, and B are differentiable functions of μ, for μ sufficiently small assumptions I and II will hold if A_f is replaced by A_{22}, and A and B are replaced by

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}, \qquad B_0 = B_1 - A_{12}A_{22}^{-1}B_2 \tag{2.11}$$

which are the matrices of the plant without parasitics,

$$\dot{\bar{x}} = A_0\bar{x} + B_0 u \tag{2.12}$$

obtained by setting $\mu = 0$ in (2.7), (2.8) and in Fig. 2.1.

Assumption III: Input $u(t)$ and its derivative $\dot{u}(t)$ are uniformly bounded.

In the subsequent sections it will be shown that the composite state/parameter error vector $Z(t)$ of several adaptive schemes is governed by the same general equation

$$\dot{Z}(t) = A_\eta(t)Z(t) + E\eta(t) + F\theta(t) + Fe^{\Lambda t}Q \tag{2.13}$$

where $\theta(t)$ satisfies

$$\dot{\theta}(t) = \Lambda\theta(t) + R\eta(t), \qquad \theta(0) = 0, \tag{2.14}$$

$E = [-H^T \vdots\ 0]^T$ and F, Q, R, and Λ are constant matrices and $\mathrm{Re}\lambda(\Lambda) < 0$. We point out that, although the homogeneous part of (2.13) is linear, this system is not input to state linear because $A_\eta(t)$ depends on x which in turn depends

on η. For this reason we introduce the notion of dominantly rich inputs in Section 2.3. We then use this notation to establish u.a.s. of the homogeneous part of (2.13) for each particular adaptive scheme. Under this condition the following lemma furnishes a bound for the state/parameter composite error Z(t).

Lemma 2.1.1: If the homogeneous part of (2.13) is u.a.s. then Z(t) is bounded by

$$\lim_{t\to\infty} \sup \|Z(t)\| \le \mu\gamma \frac{m_1}{m_2} \alpha g(1+f) \tag{2.15}$$

where

$$\gamma = \sup|\dot{u}| \tag{2.16}$$

$$g = \|A_f^{-1}\|\|A_f^{-1}B_f\|\|H\|, \quad f = \frac{\|F\|\|R\|}{\|H\|}\frac{f_1}{f_2} \tag{2.17}$$

and m_1, m_2, α, f_1, and f_2 are positive constants obtained from the state transition matrices of (2.13), A_f and Λ as follows

$$\|\phi_\eta(t,\tau)\| \le m_1 e^{-m_2(t-\tau)}, \ \|e^{A_f(t-\tau)}\| \le \alpha e^{-\beta(t-\tau)}, \ \|e^{\Lambda(t-\tau)}\| \le f_1 e^{-f_2(t-\tau)}. \tag{2.18}$$

Proof. From (2.13) and (2.18) we have

$$\|Z(t)\| \le m_1 e^{-m_2 t}\|Z(0)\| + \int_0^t m_1 e^{-m_2(t-\tau)} \|E\|\|\eta(\tau)\|d\tau$$
$$+ \int_0^t m_1 e^{-m_2(t-\tau)} \|F\|\|\theta(\tau)\|d\tau + \int_0^t m_1 e^{-m_2(t-\tau)} \|F\| f_1 e^{-f_2\tau}\|Q\|d\tau. \tag{2.19}$$

In view of (2.8), (2.10), (2.14), and (2.18) we can write

$$\|\eta(t)\| \le \alpha e^{-\beta t/\mu}\|\eta(0)\| + \int_0^t \alpha e^{-\frac{\beta(t-\tau)}{u}} \|A_f^{-1}B_f\|\gamma d\tau \tag{2.20}$$

$$\|\theta(t)\| \le f_1 \int_0^t e^{-f_2(t-\tau)} \|R\|\|\eta(\tau)\|d\tau \tag{2.21}$$

where

$$\gamma = \sup_t |\dot{u}|, \quad \beta = \min|\lambda(A_f)| \ge \|A_f^{-1}\|^{-1}, \quad f_2 = \min|\lambda(\Lambda)|. \tag{2.22}$$

Substituting for $\eta(\tau)$ and $\theta(\tau)$ in (2.19) we obtain

$$\|Z(t)\| \le p_1 e^{-m_2 t} + p_2 e^{-\beta t/\mu} + p_3 e^{-f_2 t} + \mu\gamma \frac{m_1}{m_2} \frac{\alpha}{\beta} \|A_f^{-1} B_f\| \left(\|H\| + \frac{f_1}{f_2}\|F\|\|R\|\right) \tag{2.23}$$

where p_1, p_2, and p_3 are independent of t and bounded for all $\mu \in [0,1]$. Thus as $t \to \infty$, (2.15) follows.

It is useful to emphasize the significance of the terms appearing on the bound (2.15)

$g(1+f)$ - characterizes the gain in the disturbance path

α, m_1 - characterize the decaying properties of the parasitic modes and the modes of the homogeneous part of (2.13), respectively

m_2 - the convergence rate of the homogeneous part of (2.13)

γ - a measure of the frequency content of the input signal

μ - the speed ratio of slow vs. fast phenomena.

In the subsequent sections $\mu\gamma$ will emerge as the most important factor. It is already clear from (2.15) that if Lemma 2.1.1 holds and $\gamma < 0(\frac{1}{\mu})$ then the scheme is robust in the sense that the composite error tends to zero as the frequency of parasitics tends to infinity, that is as $\mu \to 0$. However,

if $\gamma > O(\frac{1}{\mu})$ this robustness property cannot be ascertained. Further discussion of the bound (2.15) and an example are given in Section 2.5.

2.2. Dominantly Rich Inputs

A well known requirement for asymptotic identification is that the input u(t) be "rich" enough to "persistently excite" all the modes of the plant [27-29]. This is required when all the modes are to be identified. However, our goal is to identify only the dominant and to disregard the parasitic modes. In this situation inputs exciting only the dominant modes seem more appropriate. We therefore introduce the notion of "dominantly rich" inputs and in the subsequent sections we show that they guarantee the robustness of several adaptive schemes. Although our analysis is applicable to a wider class of signals, such as those defined in [29], for clarity this presentation is restricted to inputs of the form

$$u(t) = \sum_{i=1}^{k} a_i \sin \omega_i t \tag{2.24}$$

where ω_i are all distinct and positive and $a_i \neq 0$.

Definition 2.2.1: Signal (2.24) is a dominantly rich input for the plant (2.7), (2.8) if there exists $\mu^* > 0$ such that for all $\mu \in [0,\mu^*)$ the vector $[x^T,u^T]^T$ is "persistently spanning" (ps) in the sense of [29].

We point out that this definition differs from the usual sufficient richness condition for (2.7), (2.8) which requires the ps-property of the vector $[x^T,x_f^T,u^T]^T$. An important practical question is whether every signal which is sufficiently rich for the nth order plant without parasitics (2.12) is also dominantly rich for the plant with parasitics (2.7), (2.8). As the

example at the end of this section shows, the dominant richness can be lost if some frequencies in u(t) are $\geq 0(\frac{1}{\mu})$. To prevent this we restrict the frequencies in (2.24) to be lower than $0(\frac{1}{\mu})$.

Lemma 2.2.1: If an input (2.24) with $\omega_i < 0(\frac{1}{\mu})$ $i = 1,\ldots,k$ is sufficiently rich for the plant without parasitics (2.12) then it is also dominantly rich for the plant with parasitics (2.7),(2.8).

Proof: The steady state response of x to the almost periodic input (2.24) is almost periodic, that is

$$x = \sum_{i=1}^{k} c_i \sin\omega_i t + d_i \cos\omega_i t. \tag{2.25}$$

The dominant richness is achieved if $[x^T,u^T]$ is ps. For this it is sufficient [29] to show that for all $p \in R^{1\times n}$ and $q \in R^{1\times r}$

$$px + qu = 0 \Rightarrow p = 0,\ q = 0. \tag{2.26}$$

Substitution of (2.24) and (2.25) into (2.26)

$$\sum_{i=1}^{k}((pc_i + qa_i)\sin\omega_i t + pd_i \cos\omega_i t) = 0, \tag{2.27}$$

implies that

$$pc_i + qa_i = 0, \quad pd_i = 0, \quad i = 1,2,\ldots,k. \tag{2.28}$$

Following the same procedure as in [29] we use (2.28) and the relation

$$c_i + jd_i = (j\omega_i I-A)^{-1}(B + \mu H(\mu j\omega_i I-A_f)^{-1}A_f^{-1}B_f j\omega_i)a_i \tag{2.29}$$

to obtain

$$p(\omega_i^2 I + A^2)^{-1}(B-\mu AH(\mu^2\omega_i^2 I + A_f^2)^{-1}B_f+\mu^2\omega_i^2 H(\mu^2\omega_i^2 I + A_f^2)^{-1}A_f^{-1}B_f) = 0 \tag{2.30}$$

which, after grouping the $0(\mu)$ terms becomes

$$p(\omega_1^2 I + (A_o + 0(\mu))^2)^{-1}(B_o + 0(\mu) + 0(\mu^2\omega_1^2)) = 0. \tag{2.30}$$

By Lemma 4 in [29], if the pair $(A_o + 0(\mu),\ B_o + 0(\mu) + 0(\mu^2\omega_1^2))$ is controllable then (2.31) implies p=0. As long as $\omega_1 < 0(\frac{1}{\mu})$ the controllability of (A_o, B_o) guarantees that there exists $\mu^* > 0$ such that for all $\mu \in [0,\mu^*)$ the pair $(A_o + 0(\mu),\ B_o + 0(\mu) + 0(\mu^2\omega_1^2))$ is controllable [30,31]. Then $p = 0$ and hence $q = 0$ in view of (2.28).

Lemma 2.2.1 excludes input frequencies of $0(\frac{1}{\mu})$ and higher. It is of interest to illustrate that in the presence of parasitics high input frequencies can indeed destroy the dominant richness of the input. Consider

$$\dot{x} = -a_1 x + b_1 u + a_{12}\eta \tag{2.32}$$

$$\mu\dot{\eta} = -a_2\eta + \mu b_2\dot{u} \tag{2.33}$$

where $a_1, a_2, \omega > 0$. The input $u = \sin \omega t$ is sufficiently rich for the dominant part (2.32) without parasitics $(\eta = 0)$. For this input to be dominantly rich the equation

$$p\,\frac{1}{(\omega^2 + a_1^2)}\left[b_1 - \mu\,\frac{a_1 a_{12} b_2 a_2}{\mu^2\omega^2 + a_2^2} + \mu^2\,\frac{\omega^2 a_{12} b_2}{(\mu^2\omega^2 + a_2^2)}\right] = 0 \tag{2.34}$$

should imply $p = 0$. However this is not so since a frequency ω zeroing the expression in the brackets can be found. For example, if $a_1 = 1$, $a_{12} = 1$, $a_2 = 1$, $b_2 = -1$, $b_1 = 0.5$ then given any $\mu \in [0,\mu^*)$ there exists an $\omega \geq 0(\frac{1}{\mu})$ such that $\mu^2\omega^2 = 1+2\mu$ and the term in the brackets is zero. For this reason the input frequencies are restricted to $\omega < 0(\frac{1}{\mu})$.

Let us now show that the dominant richness of u assures the robustness of the simple identification scher.e [2],[29]. The problem is to identify the dominant part of the plant, that is the pair (A,B) in (2.7),

when the dominant state x is available for measurement. We disregard the presence of the parasitic input $H\eta$ in the design of the identification algorithm, that is we assume the nth order model

$$\dot{x}_m = K(x_m - x) + A_m(t)x + B_m(t)u \tag{2.35}$$

where K is a stable matrix. As in [2] the adaptive laws for adjusting $A_m(t)$ and $B_m(t)$ are

$$\dot{\phi} = -\Gamma e x^T \tag{2.36}$$

$$\dot{\Psi} = -\Gamma e u^T \tag{2.37}$$

where $\phi \triangleq A_m(t) - A$, $\Psi \triangleq B_m(t) - B$, and $e \triangleq x_m - x$ are the parameter and state errors, respectively, and $\Gamma = \Gamma^T > 0$. When the adaptive algorithm (2.35) to (2.37) is applied to the actual plant (2.7), (2.8), then the following set of equations describe the behavior of the identification scheme in the presence of parasitics

$$\dot{e} = Ke + \phi x + \Psi u - H\eta \tag{2.38}$$

$$\dot{\phi} = -\Gamma e x^T \tag{2.39}$$

$$\dot{\Psi} = -\Gamma e u^T. \tag{2.40}$$

Defining $Z(t) = [e^T, \chi^T, T^T]$ where $\chi = [\phi_1, \phi_2, \ldots, \phi_n]^T$, $T = [\Psi_1, \Psi_2, \ldots, \Psi_n]^T$, ϕ_i, Ψ_i are the ith rows of ϕ and Ψ, respectively, and denoting

$$A_\eta(t) = \left[\begin{array}{c|c|c} K & \begin{matrix} x^T & & 0 \\ & \ddots & \\ 0 & & x^T \end{matrix} & \begin{matrix} u^T & & 0 \\ & \ddots & \\ 0 & & u^T \end{matrix} \\ \hline -\Gamma_x & & \\ \cline{1-1} -\Gamma_u & \multicolumn{2}{c}{0} \end{array}\right], \quad \bar{H} = \left[\begin{array}{c} -H \\ \hline 0 \end{array}\right] \tag{2.41}$$

$$\Gamma_x = \begin{bmatrix} x\gamma_1 \\ x\gamma_2 \\ \vdots \\ x\gamma_n \end{bmatrix}, \qquad \Gamma_u = \begin{bmatrix} u\gamma_1 \\ u\gamma_2 \\ \vdots \\ u\gamma_n \end{bmatrix} \tag{2.42}$$

where γ_j is the jth row of Γ, equations (2.38) to (2.40) become (2.13) with F=0. To analyze the stability of the identification scheme in the presence of parasitics we first establish the u.a.s. of

$$\dot{\bar{Z}}(t) = A_\eta(t)\bar{Z}(t) \tag{2.43}$$

the homogeneous part of (2.13). Although (2.43) is formally similar to the error equation for the nth order system without parasitics, the crucial difference is that $A_\eta(t)$ in (2.43) depends on η. For this reason the u.a.s. of (2.43) does not immediately follow from [2],[29] and is established here.

<u>Theorem 2.2.1</u>: If u(t) is dominantly rich for the system (2.7), (2.8) then (2.43) is u.a.s. and by Lemma 2.1.1 the composite state/parameter error Z(t) is bounded by

$$\lim_{t\to\infty} \sup \|Z(t)\| \leq \mu\gamma \frac{m_1}{m_2} \alpha g. \tag{2.44}$$

<u>Proof</u>: Define

$$P = \begin{bmatrix} \Gamma & 0 \\ 0 & I \end{bmatrix} > 0, \qquad Q = \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \tag{2.45}$$

where $M = -(K^T\Gamma + \Gamma K) > 0$. Since the derivative of

$$V(\bar{Z}) = \bar{Z}^T P \bar{Z} \tag{2.46}$$

for (2.43) is

$$\dot{V}(\bar{Z}) = -\bar{Z}^T Q \bar{Z} = -\bar{e}^T M \bar{e} \tag{2.47}$$

system (2.43) is uniformly stable. From (2.47) and (2.43) we conclude that $e \to 0$, $\dot{\bar{Z}} \to 0$ as $t \to \infty$ and (2.43) reduces to

$$\bar{\phi}_i x + \bar{\Psi}_i u = 0, \qquad i = 1,2,\ldots,n \tag{2.48}$$

where $\bar{e}$, $\bar{\phi}_i$, and $\bar{\Psi}_i$ are the corresponding values of e, ϕ_i, and Ψ_i associated with the homogeneous part (2.43).

In Lemma 2.2.1 we have established that when u(t) is dominantly rich, then $[x^T, u^T]$ is ps. Thus (2.48) implies $\bar{\phi}_i = 0$, $\bar{\Psi}_i = 0$, i=1,2,...,n which proves that (2.48) is u.a.s. Once u.a.s. of the homogeneous part (2.43) is established Lemma 2.1.1 furnishes the bound (2.44).

In conclusion, the identification of the dominant part, which is a type of reduced order identification, differs from the full order identification [2],[29] in the dependence of x and hence $A_\eta(t)$ on η and the presence of the forcing term $H\eta$ in (2.38).

Furthermore, instead of the full state, only the dominant state appears in the steady state equation (2.48). The dominant richness condition, which avoids excessive excitation of the neglected fast modes, guarantees the u.a.s. However, the asymptotic identification is not achieved since the forcing term $H\eta$ produces a steady-state parameter error. The identification is robust in the sense that the parameter error is $O(\mu)$ and tends to zero as $\mu \to 0$, that is as parasitics disappear.

2.3. Reduced Order Adaptive Observers

If instead of the dominant state x only the output is available for measurement, the problem of identifying the parameters and estimating the dominant state x for the plant (2.7), (2.8) simultaneously is a reduced order adaptive observer problem. Here we assume that the output is of the form

$$y = C^T x \tag{2.49}$$

that is the parasitics are only weakly observable from (2.49) since the dependence of y on the parasitic modes is $O(\mu)$. The case of strongly observable parasitics, $y = C_1 x + C_2 x_f$, requires a different type of analysis and is postponed until Section 4.

We assume that several different nth order adaptive observers [32-34] for SISO systems have been designed to identify the triple (A,B,C) and estimate the dominant state x of (2.7), (2.8) by disregarding the parasitics. Then we investigate the behavior of these schemes when applied to the actual plants with parasitics.

We start with two different types of Minimal Form Adaptive Observers [32],[33]. We have shown in Appendix A that their stability properties in the presence of parasitics are determined by

$$\dot{\varepsilon} = K\varepsilon + d(\phi^T v + \Psi^T q) - H\eta \tag{2.50}$$

$$\varepsilon_1 = e_1 \tag{2.51}$$

$$\dot{\phi} = -\Gamma\varepsilon_1 v \tag{2.52}$$

$$\dot{\Psi} = -M\varepsilon_1 q \tag{2.53}$$

where ϕ, Ψ are the parameter identification errors, e_1 is the output error, ε is a function of the observation error and v,q are the filtered output and input signals respectively. To apply Lemma 2.1.1 we rewrite (2.50) to (2.53) in the form of (2.13) with $F=0$ by defining

$$Z(t) = [\varepsilon^T, \phi^T, \psi^T]^T \tag{2.54}$$

$$A_\eta(t) = \left[\begin{array}{c|c|c} K & dv^T & dq^T \\ \hline -\Gamma v & & \\ & 0 & \\ -Mq & & \end{array}\right]. \tag{2.55}$$

Then the uniform stability of the homogeneous part of (2.13) can be shown by using the same Lyapunov function as in the case without parasitics [32],[33] and it is not influenced by the fact that $A_\eta(t)$ depends on v, which in turn depends on η through x_1. A sufficient condition for u.a.s. of the homogeneous part of (2.13) is that $[v^T, q^T]$ be ps. As in Lemma 2.2.1, if u(t) is dominantly rich then $[v^T, q^T]$ is ps and therefore the homogeneous part of (2.13) is u.a.s. This result is stated as follows.

<u>Corollary 2.3.1</u>: If u is dominantly rich then the homogeneous part of (2.13) is u.a.s. Furthermore the composite error vector Z(t) of (2.13) is bounded by

$$\lim_{t\to\infty} \sup \| Z(t) \| \leq \mu\gamma \frac{m_1}{m_2} \alpha g. \tag{2.56}$$

Next we consider a <u>Non-minimal Adaptive Observer</u> [2],[33] whose behavior in the presence of parasitics is analyzed in Appendix B and is described by

$$\dot{e}_1 = -\lambda_1 e_1 + d(\phi^T v + \Psi^T q) - h^T\theta - h^T e^{\Lambda t}\bar{x}(0) \tag{2.57}$$

$$\dot{\phi} = -\Gamma e_1 v \tag{2.58}$$

$$\dot{\Psi} = -M e_1 q \tag{2.59}$$

where e_1 is the output error, ϕ, Ψ are the parameter identification error, v and q are functions of the states and the input of the non-minimal representation of the plant and θ satisfies (2.14). Defining

$$Z(t) = [e_1, \phi^T, \Psi^T]^T \tag{2.60}$$

$$A_\eta(t) = \left[\begin{array}{c|c|c} -\lambda_1 & dv^T & dq^T \\ \hline -\Gamma v & & \\ & \multicolumn{2}{c}{0} \\ -Mq & & \end{array}\right], \quad F = \begin{bmatrix} -h^T \\ \hline \\ 0 \end{bmatrix}, \quad Q = \begin{bmatrix} \bar{x}(0) \\ \hline \\ 0 \end{bmatrix} \tag{2.61}$$

we rewrite (2.57) to (2.59) in the form (2.13) with E=0 and, similarly, as before, obtain the following result.

<u>Corollary 2.3.2</u>: If the input signal is dominantly rich the homogeneous part of (2.13) is u.a.s. and the composite error Z(t) in (2.13) is bounded by

$$\lim_{t\to\infty} \sup \|Z(t)\| \le \mu\gamma \frac{m_1}{m_2} \alpha g_f \tag{2.62}$$

where $g_f = \|A_f^{-1}\| \|A_f^{-1}B_f\| \|h\| \|R\| \dfrac{f_1}{f_2}$.

Finally we analyze the <u>Parameterized Observer</u> [34] whose state observation error e and parameter error Δp is shown in Appendix C to satisfy

$$\dot{e} = -M(t)GM^T(t)CC^T e + (\Lambda M(t) + (Iy, Iu))\Delta p - \Lambda e^{\Lambda t} e(0) - \Lambda\theta - H\eta \tag{2.63}$$

$$\Delta\dot{p} = -GM^T(t)CC^T e. \tag{2.64}$$

Combining (2.63) and (2.64) the equation for the composite error $Z(t) = [e^T, \Delta p^T]^T$ is of the form of (2.13) where $F^T = [-\Lambda^T \; 0^T]$, $Q^T = [e^T(0), 0^T]$, $R = H$ and

$$A_\eta(t) = \begin{bmatrix} -M(t)GM^T(t)CC^T & \Lambda M(t) + [Iy, Iu] \\ -GM^T(t)CC^T & 0 \end{bmatrix}. \tag{2.65}$$

In this case the dominant richness appears indirectly as a sufficient condition for

$$0 < k_1 I \le \int_t^{t+T} M^T(\tau)CC^T M(\tau)d\tau \le k_2 I \quad \text{for all } t \ge 0. \tag{2.66}$$

Constants k_1, k_2, and T rate the degree of linear independence of the components of the vector $M^T(t)C$. In the absence of parasitics a sufficient condition for existence of k_1, k_2, and T is that u is a sum of sinusoids with at least n distinct frequencies [34]. Then $[x^T, u]$ is ps and therefore the components of $M^T(t)C$ are linearly independent. In the presence of parasitics this nth order richness of u and hence the linear independence of $M^T(t)C$ can be destroyed. To prevent this we assume that the input is dominantly rich.

<u>Theorem 2.3.1</u>: If the input u is dominantly rich then the homogeneous part of (2.13) is u.a.s. and Z is bounded by

$$\lim_{t\to\infty} \sup \|Z\| \le \mu\gamma \frac{m_1}{m_2} \alpha g(1+f) \tag{2.67}$$

where $$f = \frac{f_1}{f_2}\|\Lambda\| \quad \text{and} \quad m_2 = \frac{k_1 \min \lambda[G]}{(1 + nk_2 \max \lambda[G])^2}.$$

Proof: Using the same proof as in Theorem 1 of [34] we can show that if there exists constants k_1, k_2, and T such that (2.66) is satisfied then the homogeneous part of (2.13) is u.a.s. with a rate of convergence no less than m_2 where

$$m_2 = \frac{k_1 \min \lambda[G]}{(1 + nk_2 \max \lambda[G])^2}. \tag{2.68}$$

Once the u.a.s. of the homogeneous part of (2.13) is established Lemma 2.1.1 furnishes the bound (2.67).

2.4. Discussion and Example

Adaptive schemes designed for plants without parasitics, but applied to plants with parasitics are considered robust if the error due to parasitics is $O(\mu)$. This means that, first, the error is bounded, and second, it tends to zero as parasitics vanish, i.e. as $\mu \to 0$. Among the factors common to all the error bounds derived in this paper,

$$\mu\gamma \frac{m_1}{m_2} \alpha g, \tag{2.69}$$

factor $\mu\gamma$ determines the robustness property. If, by a choice of high frequency input, $\gamma = \sup|\dot{u}|$ is made $O(\frac{1}{\mu})$ or larger, then $\mu\gamma \geq O(1)$ and the error bounds do not tend to zero as $\mu \to 0$. This will be the case if u(t) contains terms $\sin \frac{\sigma}{\mu} t$ where σ is a constant, that is if input frequencies are in the range of parasitic frequencies. For such inputs (2.69) does not

exclude the possibility that the state/parameter errors may be significant. In fact if u contains frequencies of $O(\frac{1}{\mu})$ the fast state η will be of $O(1)$, $H\eta$ will be persistent for all μ and therefore robustness will be lost. Our results also show that the presence of parasitics may cause loss of richness, if the richness is achieved with frequencies in the parasitic range. On the other hand the dominantly rich inputs guarantee the robustness property.

Let us illustrate this and other aspects of the derived error bounds using the plant

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} .9 \\ .5 \end{bmatrix} x_f + \begin{bmatrix} 1.45 \\ 2.25 \end{bmatrix} u \tag{2.70}$$

$$\mu\dot{x}_f = -4x_f - 2u \tag{2.71}$$

$$y = [1,0]x \tag{2.72}$$

which, upon the transformation $\eta = x_f + 0.5u$ becomes

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u + \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix} \eta \tag{2.73}$$

$$\mu\dot{\eta} = -4\eta + \mu 0.5\dot{u} \tag{2.74}$$

$$y = [1,0]x. \tag{2.75}$$

Note that this is an example of the plant (2.1), (2.2), (2.49) with $A_{21} = 0$ and hence $L = 0$, $A_{22} = A_f$, $A_{11} = A$. Suppose that we neglect the parasitics and design a minimal form adaptive observer [32] for the dominant plant (2.73), (2.75) with $\eta = 0$, that is

$$\dot{x} = \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix} x + \begin{bmatrix} 6 - a_1(t) \\ 8 - a_2(t) \end{bmatrix} y + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} u - e_1 \begin{bmatrix} 0 \\ -420v_1v_2 + 75v_2^2 \end{bmatrix} - e_1 \begin{bmatrix} 0 \\ -420v_1v_2 + 75v_2^2 \end{bmatrix} \tag{2.76}$$

$$\dot{\hat{a}}_1(t) = 140 v_1 e_1, \quad \dot{\hat{a}}_2(t) = 75 v_2 e_1, \quad \dot{\hat{b}}_1(t) = -5 q_1 e_1, \quad \dot{\hat{b}}_2(t) = -7.8 q_2 e_1 \tag{2.77}$$

$$\dot{v}_2 = -3v_2 + x_1 = v_1, \quad \dot{q}_2 = -3q_2 + u = q_1 \tag{2.78}$$

where $\hat{x}$ is an estimate of x and $\hat{a}_i(t)$, $\hat{b}_i(t)$ $i = 1,2$ are the estimates of the unknown parameters. The adaptive observer (2.76) to (2.78) is now applied to the actual system (2.73) to (2.75) with parasitics and (2.73) to (2.78) are simulated on a digital computer.

The dependence of the error bound on μ is illustrated in Figs. 2.2 and 2.3. For $\mu = 0.2$ and input $u = 5 \sin 2.5t$ that is $\gamma = 18.2$, the observation error e_2 is relatively small. However, the parameter errors are significant: 10.4% for $\hat{a}_1(t)$ and 12% for $\hat{b}_1(t)$. Reduction of μ by a factor 4, that is $\mu = 0.05$, results in a reduction of the parameter errors by approximately the same factor as shown in Fig. 2.3b,c. The observation error e_2 is almost zero in this case (Fig. 2.3a). Thus for a fixed frequency the parameter error is $O(\mu)$ as predicted by (2.69).

To examine the effect of $\gamma = \sup|\dot{u}|$ on the error bound the value of μ is kept the same as in Fig. 2.3, but the amplitude of the higher frequency is increased, $u = 5 \sin t + 15 \sin 2.5t$, that is γ is increased to $\gamma = 42.5$. As shown in Fig. 2.4 increasing γ by a factor of 2.3 results in an increase of the parameter error by a factor of about 10. Moreover, the observation error, although bounded, is oscillatory and not close to zero.

An even more critical way to change the frequency content of the input signal is to increase the higher frequency $\omega = 2.5$ tenfold to $\omega = 25$, $u = 5 \sin t + 5 \sin 25t$. Decreasing the value of μ by the same factor, from $\mu = 0.2$ to $\mu = 0.02$, does not reduce the identification error which in Fig. 2.5

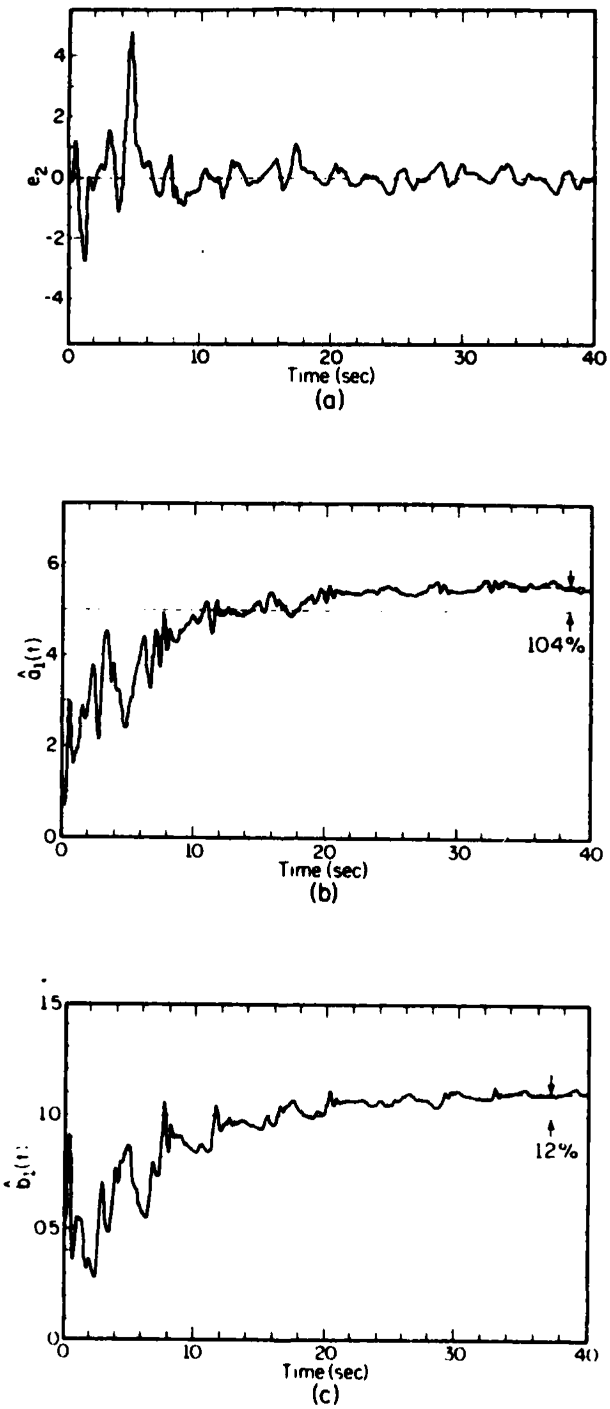


Fig. 2.2 Identification results for $\mu = 0.2$ and $u = 5\sin t + 5\sin 2.5t$ ($\gamma = 18.2$).

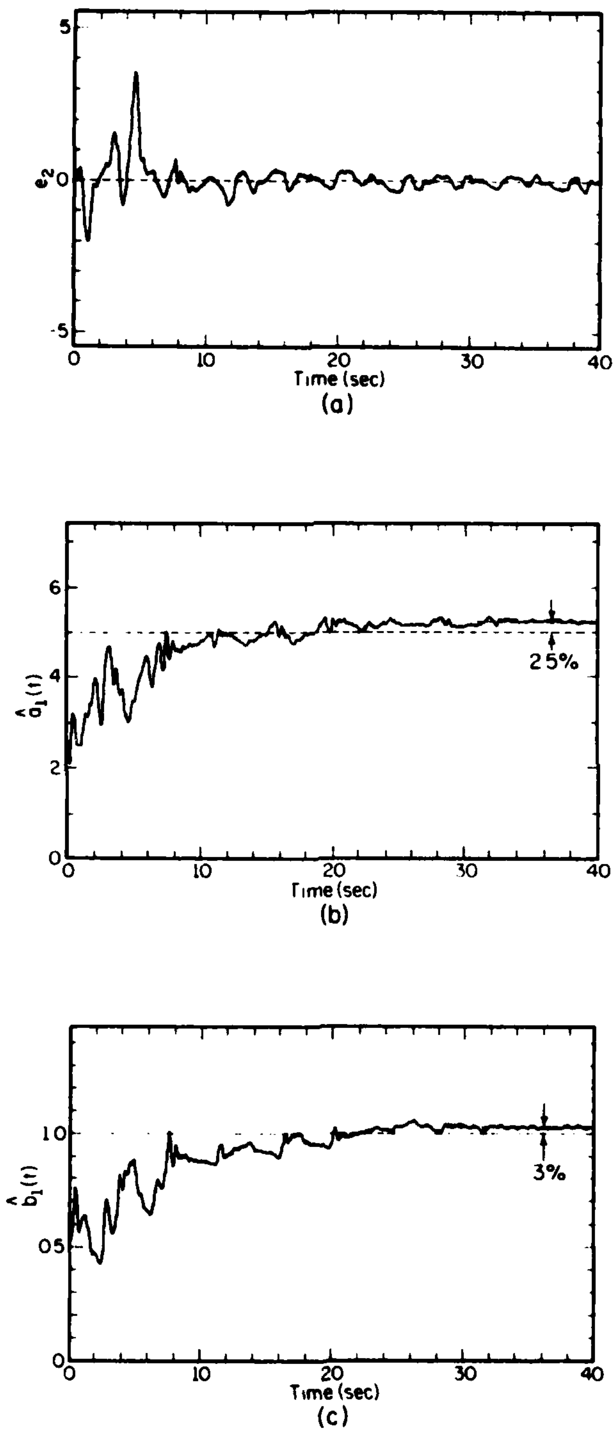


Fig. 2.3 Identification results for μ = 0.05 and u = 5sint + 5sin2.5t (γ = 18.2).

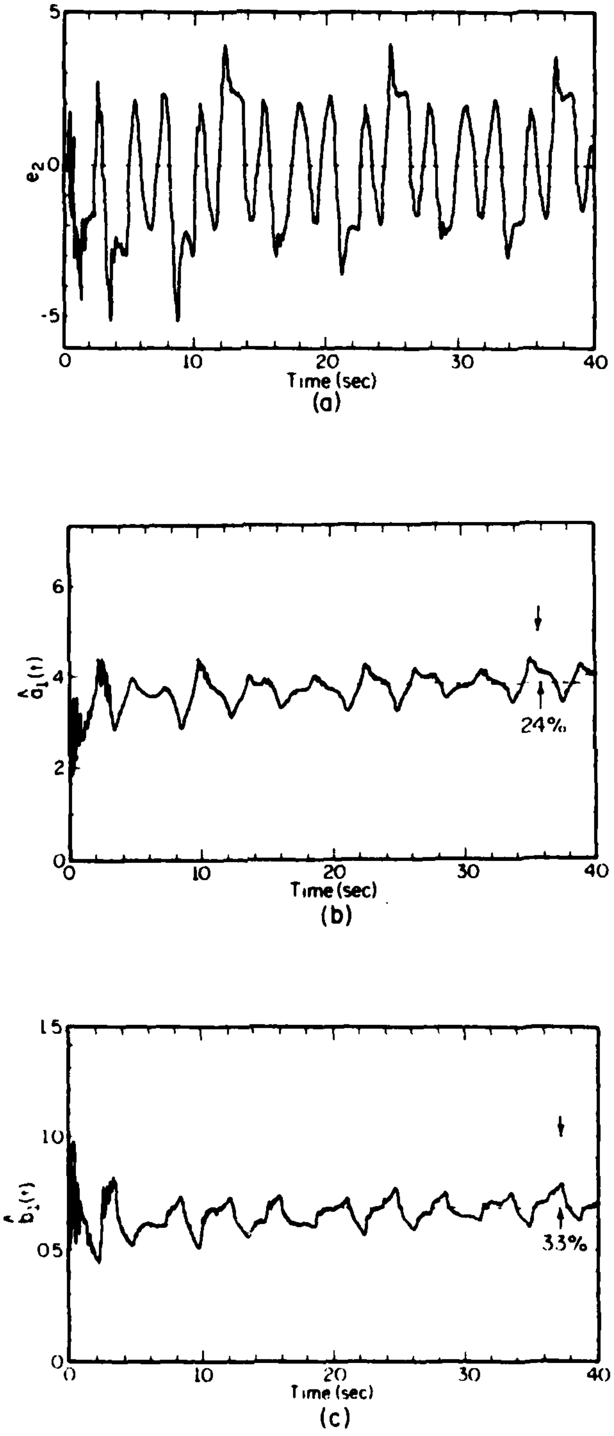


Fig. 2.4 Identification results for $\mu = 0.05$ and $u = 5\sin t + 15\sin 2.5t$ ($\gamma = 42.5$).

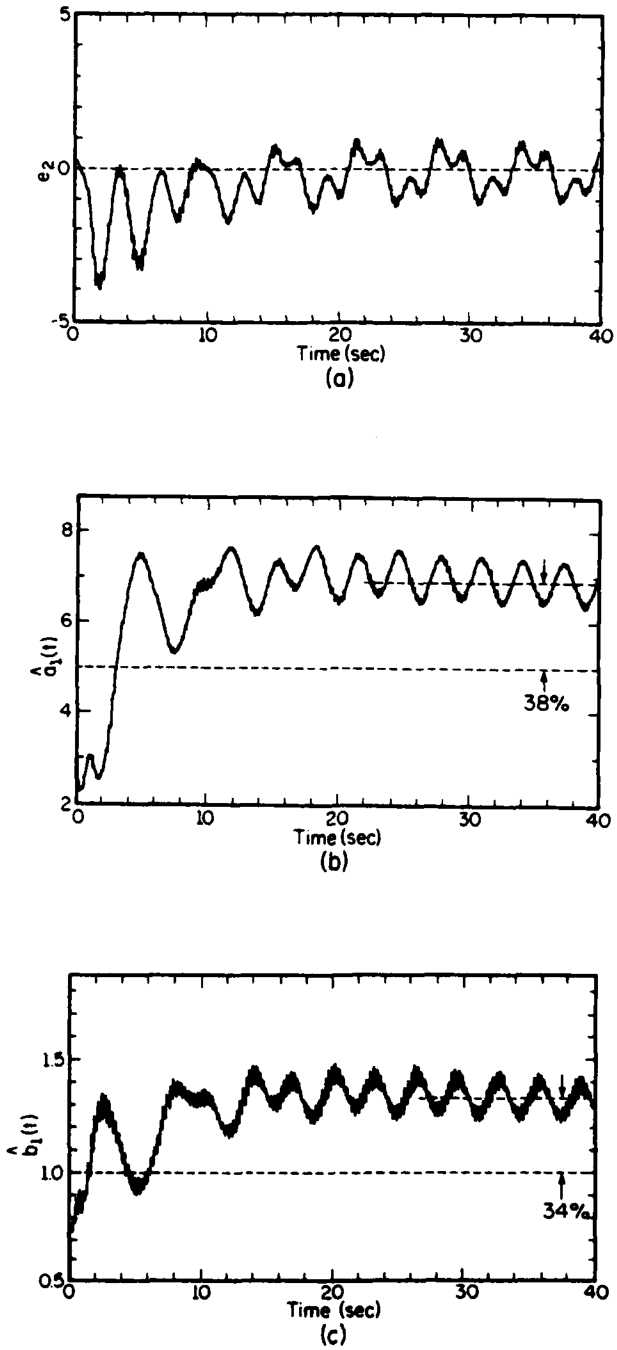


Fig. 2.5 Identification results for μ = 0.02 and u = 5sint + 5sin25t (γ = 130).

is about 3 times higher than in Fig. 2.2. This illustrates that input frequencies of $O(\frac{1}{\mu})$ may cause state/parameter errors which persist or even increase as $\mu \to 0$. Hence loss of robustness and a possibility of instability.

A trade off in selecting the input is apparent from this discussion. High frequencies are to be avoided since they excite the parasitic modes. On the other hand low frequencies affect the convergence rate adversely. It is appropriate to use a dominantly rich input with a low value of γ still giving an acceptable rate of convergence.

The improvement of the bound by making the convergence rate m_2 larger may require trial and error selection of the adaptive gains. An exception is the parameterized adaptive observer where the expression

$$m_2 = \frac{k_1 \min \lambda[G]}{(1 + nk_2 \max \lambda[G])^2} \tag{2.79}$$

gives more information about the dependence of the error bound on other quantities. In this case m_2 is the largest when $\max \lambda[G] = \min \lambda[G]$. Note from (2.79) that the error bound will be higher when the order n of the dominant part of the plant is higher.

3. DISCRETE-TIME PARALLEL AND SERIES PARALLEL IDENTIFIERS

3.1. Characterization of the Mismatch

A class of discrete systems possessing a two-time scale property has the form [35]

$$x(k+1) = A_{11}x(k) + \mu A_{12}z(k) + B_1u(k) \tag{3.1}$$

$$z(k+1) = A_{21}x(k) + \mu A_{22}z(k) + B_2u(k) \tag{3.2}$$

$$y(k) = C_1x(k) + \mu C_2z(k) \tag{3.3}$$

where A_{11}^{-1} exists and μ is a small positive parameter. The states $x(k)$ and $z(k)$ are n and m vectors respectively and $u(k)$ and $y(k)$ are the scalar input and output respectively. The restriction of the output of the model to be of the form (3.3) makes the parasitics weakly observable in the plant output. This weak observability assumption is found to be crucial in estabilishing the robustness of the adaptive schemes, i.e. as $\mu \to 0$ (modeling error $\to 0$) the composite identification error $\to 0$. The case of strongly observable parasitics requires a different kind of treatment and it will be discussed in Section 4. We use the tranformation

$$\eta(k) = z(k) + Px(k) - B_fu(k) \tag{3.4}$$

to obtain the representation (3.5) to (3.7)

$$x(k+1) = A_sx(k) + B_su(k) + \mu H\eta(k) \tag{3.5}$$

$$\eta(k+1) = \mu A_f\eta(k) + B_f[u(k)-u(k+1)] \tag{3.6}$$

$$y(k) = C_sx(k) + \mu C_2B_fu(k) + \mu C_2\eta(k) \tag{3.7}$$

where P satisfies

$$A_{21} + PA_{11} - \mu A_{22}P - \mu PA_{12}P = 0 \tag{3.8}$$

and $H = A_{12}$, $A_s = A_{11} - \mu A_{12}P$, $B_s = B_1 + \mu A_{12}B_f$, $C_s = C_1 - \mu C_2P$, $A_f = A_{22} + PA_{12}$, $B_f = (I - \mu A_f)^{-1}(B_2 + PB_1)$. Representation (3.5) to (3.7) is found to be convenient for characterizing the error bounds of the adaptive schemes considered. Without loss of generality let us assume that the model of the dominant part of the plant is in the observable canonical form, i.e.

$$A_s = \begin{bmatrix} a_1 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & & & \vdots \\ \vdots & \vdots & & \ddots & & 0 \\ \vdots & \vdots & & & \ddots & 1 \\ a_n & 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \quad C_s = [1 \quad 0 \; \ldots \; 0]$$

and obtain the following representation for the plant

$$x_s(k+1) = A_s x_s(k) + b_s u(k) + \mu H_s \eta(k) \tag{3.9}$$

$$\eta(k+1) = \mu A_f \eta(k) + B_f[u(k) - u(k+1)] \tag{3.10}$$

$$y(k) = [1 \quad 0 \quad \ldots \quad 0] x_s(k) = C_s^T x_s(k) \tag{3.11}$$

also shown in Fig. 3.1

where $b_s = B_s - \mu a_s C_2 B_f + \mu[C_2B_f, 0\ldots0]^T$, $H_s = H - a_s C_2 + \mu[C_2A_f, 0\ldots0]^T$

Due to this observable canonical form we can write (3.9) to (3.11) as

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n} b_i u(k-i) + y_u(k) \tag{3.12}$$

where $y_u(k)$ is the unmodeled part of the plant and is given by

$$y_u(k) = \mu \sum_{i=1}^{n} h_i \eta(k-i). \tag{3.13}$$

and h_i is the ith row of H_s.

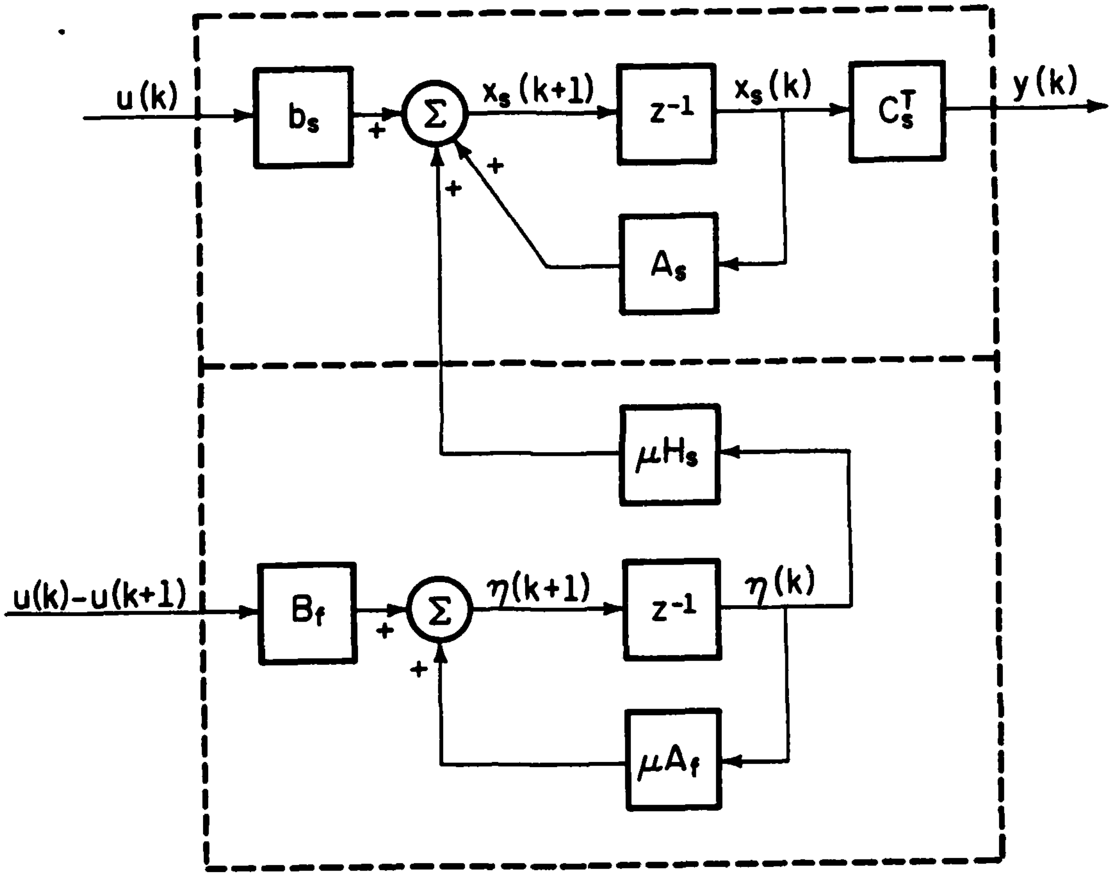


Fig. 3.1. Representation of two-time scale plant as dominant portion with parasitic input (see equations (3.9) to (3.11)).

Assume now that an identifier is designed for the nth-order dominant part to identify the vectors $a_s = [a_1 \ldots a_n]^T$, $b_s = [b_1 \ldots b_n]^T$ under the assumption that there are no parasitics, i.e. $\eta(k) = 0$. This identifier is then applied to the actual system with parasitics. Our purpose is to examine the robustness of "parallel" and "series parallel" identifiers in the presence of the parasitic input, obtain bounds for their parameter and output errors, and thereby examine their performances. Since the unknown parameters a_s, b_s are functions of μ to each value of μ there corresponds a different parameterization.

The following assumptions are made throughout this section:

(A1) A_s and A_f are stable.

(A2) The order n of the dominant part of the plant is known.

(A3) The triple (A_s, b_s, C_s) is both completely controllable and completely observable.

(A4) The only available signals are the input $u(\cdot)$ and output $y(\cdot)$.

(A5) $u(k)$ and $u(k)-u(k+1)$ (and therefore $y(k)$ due to (A1)) are bounded for all k.

3.2. Reduced Order Parallel Adaptive Identifier

Equations for an nth-order parallel identifier [6] are summarized below. The estimation model (or parallel system) is

$$y_p(k) = \sum_{i=1}^{n} \hat{a}_i(k) y_p(k-i) + \sum_{i=1}^{n} \hat{b}_i(k) u(k-i) = \hat{p}_p^T(k) \theta_p(k-1) \tag{3.14}$$

$$y_p^o(k) = \hat{p}_p^T(k-1)\theta_p(k-1) \tag{3.15}$$

where

$$\hat{p}_p^T(k) = [\hat{a}_1(k) \ \dots \ \hat{a}_n(k) \quad \hat{b}_1(k) \quad \dots \quad \hat{b}_n(k)], \tag{3.16}$$

$$\theta_p(k-1) = [y_p(k-1) \ \dots \ y_p(k-n) \quad u(k-1) \ \dots \ u(k-n)]^T, \tag{3.17}$$

and $y_p(k)$, $y_p^o(k)$ are the a posteriori and a priori output of the estimation model, respectively, at the instant k. The integral adaptation algorithm is [6]

$$\hat{p}_p(k) = \hat{p}_p(k-1) + \frac{F_p(k-1)\theta_p(k-1)\nu_p^o(k)}{1+\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)} \tag{3.18}$$

$$\nu_p^o(k) = y(k) - \hat{p}_p^T(k-1)\theta_p(k-1) + \sum_{i=1}^{n} c_i[y(k-i)-y_p(k-i)], \tag{3.19}$$

where $F_p(k-1)$ can be a decreasing positive definite matrix given by

$$F_p(k) = F_p(k-1) - \frac{F_p(k-1)\theta_p(k-1)\theta_p^T(k-1)F_p(k-1)}{\lambda+\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)}, \quad \lambda > 0.5 \tag{3.20}$$

where $F_p(k_o)$ is an arbitrary positive definite matrix. The constants $c_1,\dots,c_n$ are preselected such that

$$H(z) = \frac{1 + \sum_{i=1}^{n} c_i z^{-i}}{1 - \sum_{i=1}^{n} a_i z^{-i}} - \frac{1}{2\lambda} \tag{3.21}$$

is strictly positive real, i.e. $\mathrm{Re}\{H(z)\} > 0, \ \forall |z| = 1$.

<u>Theorem 3.2.1</u>: With u(k) satisfying a persistently exciting condition [36], the adaptive identifier (3.14) to (3.21) is stable in the presence of a parasitic input (3.13) in the sense that the composite error

$$Z_p(k) = [e_p^T(k) \quad \Delta p_p^T(k)]^T \tag{3.22}$$

is bounded for all k and

$$\lim_{k\to\infty} \sup\| Z_p(k)\| \le S(Z(k_o),\rho)\mu n\gamma h\| B_f\| \left[\frac{\alpha_1}{1-\mu\alpha_2}\right], \tag{3.23}$$

where

$$e_p(k) = [y(k)-y_p(k),\ y(k-1)-y_p(k-1),\ldots,y(k-n+1)-y_p(k-n+1)]^T, \tag{3.24}$$

$$\Delta p_p(k) = [a_1-\hat{a}_1(k),\ a_2-\hat{a}_2(k),\ldots,a_n-\hat{a}_n(k),\ b_1-\hat{b}_1(k),\ldots,b_n-\hat{b}_n(k)]^T \tag{3.25}$$

$S(Z(k_o),\rho)$ is a bounded function of the initial output and parameter estimate error $Z(k_o)$ and the exponential convergence rate ρ of (3.18) to (3.20) in the absence of the parasitic input y_u, γ is $\sup_i |u(i)-u(i+1)|$, h is $\sup_i\| h_i\|$, and α_1 and α_2 in $\alpha_1(\alpha_2)^{k-k_o}$ bound the k_o to k transition matrix of the parasitic dynamics in (3.10) with α_2 the maximum eigenvalue of A_f.

<u>Proof</u>: The behavior of the adaption algorithm of (3.14) to (3.20) in identifying (3.12) in the presence of modeling error is analyzed in Appendix D and is described by the following error system equation

$$Z_p(k+1) = A_p(k)Z(k) + B_p(k)y_u(k+1), \tag{3.26}$$

$$A_p(k) = \begin{bmatrix} A - \dfrac{\theta_p^T(k)F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\, b(a+c)^T & \dfrac{b\theta_p^T(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)} \\[2ex] \dfrac{-F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\,(a+c)^T & I - \dfrac{F_p(k)\theta_p(k)\theta_p^T(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)} \end{bmatrix} \tag{3.27}$$

$$B_p(k) = \begin{bmatrix} \dfrac{b}{1+\theta_p^T(k)F_p(k)\theta_p(k)} \\[2ex] \dfrac{-F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)} \end{bmatrix}, \tag{3.28}$$

$$a = [a_1\ a_2\ \dots\ a_n]^T, \quad b = [1\ \ 0\ \ \dots\ \ 0]^T, \quad c = [c_1\ c_2\ \dots\ c_n]^T, \tag{3.29}$$

$$A = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots\ 0 & 1 & 0 \end{bmatrix} \tag{3.30}$$

Note that (3.26) is not linear as it appears since $A_p(k)$ depends on $\theta_p(k)$ which includes $y_p(k)$ which is in $Z_p(k)$. As in [21] define $\tilde{A}_p(k)$ to be $A_p(k)$ with $\theta_p(k)$ replaced by

$$\theta(k) = [y(k), y(k-1), \dots, y(k-n+1), u(k-1), \dots, u(k-n+1)] \tag{3.31}$$

and F_p replaced by $\tilde{F}_p(k)$ with $\theta_p(k)$ replaced by $\theta(k)$ and set

$$\psi(Z_p(k), k) = A_p(k) - \tilde{A}_p(k). \tag{3.32}$$

Note that $\tilde{A}_p(k)$ is independent of $Z_p(k)$ and that

$$\theta(k) = \theta_p(k) + [e_p^T(k)\quad 0]^T. \tag{3.33}$$

Then (3.26) can be rewritten as

$$Z_p(k+1) = \tilde{A}_p(k) Z_p(k) + \psi(Z_p(k), k) Z_p(k) + B_p(k) y_u(k+1). \tag{3.34}$$

Since A_p is identical to F in [36] with $\lambda = \infty$* and B_p is bounded, then if u is sufficiently rich (in order to insure the exponential stability $Z_p(k+1) = A_p(k) Z_p(k)$ [36]) then

---

*With the retention of positive definiteness and invertibility by $F_p(k)$ in (3.20), the time-variations of $F_p(k)$ do not violate the uniform observability requirements of [36] nor, given the global asymptotic stability proven in [6] for (3.20), the conclusion of exponential convergence. Note $F_p(k)$ does not retain positive definiteness for all λ, e.g. $\lambda = 1$ even given sufficient excitation period.

$$\lim_{k\to\infty} \sup \| Z_p(k) \| < S(Z_p(k_o),\rho) \lim_{k\to\infty} \sup |y_u(k+1)| , \tag{3.35}$$

where $S(Z_p(k_o),\rho)$ is a bounded function of the initial parameter and output estimate error $Z_p(k_o)$ and the exponential convergence rate ρ in the absence of parasitic inputs.

From (3.10)

$$\eta(k) = \phi(k,k_o)\eta(k_o) + \sum_{i=k_o}^{k-1} \phi(k,i+1)B_f[u(i)-u(i+1)], \tag{3.36}$$

where $\phi(k,j)$ is the transition matrix of (3.10), i.e. $(\mu A_f)^{k-j}$. Since A_f is stable, [37]

$$\| \phi(k,j) \| \leq \alpha_1 (\mu\alpha_2)^{k-j} \text{ for } \infty > \alpha_1 > 0 \text{ and } 1 > \alpha_2 > 0. \tag{3.37}$$

Therefore, in (3.36),

$$\| \eta(k) \| \leq \alpha_1 (\mu\alpha_2)^{k-k_o} \| \eta(k_o) \| + \sum_{i=k_o}^{k-1} \alpha_1 (\mu\alpha_2)^{k-i-1} \| B_f \| \, |u(i)-u(i+1)| . \tag{3.38}$$

Defining

$$\gamma = \sup_i |u(i)-u(i+1)| \tag{3.39}$$

converts (3.38) to

$$\| \eta(k) \| \leq \alpha_1 (\mu\alpha_2)^{k-k_o} \| \eta(k_o) \| + \frac{\alpha_1}{1-\mu\alpha_2} \| B_f \| \gamma . \tag{3.40}$$

From (3.13)

$$|y_u(k+1)| \leq \mu n (\sup_{n \geq i \geq 1} \| h_i \|) (\sup_{n \geq i \geq 1} \| \eta(k-i+1) \|) . \tag{3.41}$$

Defining

$$h = \sup_{n \geq i \geq 1} \| h_i \| , \tag{3.42}$$

then, since $0 < \alpha_2 < 1$,

$$\lim_{k\to\infty} \sup |y_u(k+1)| \leq \mu n \gamma h \| B_f \| \left[\frac{\alpha_1}{1-\mu\alpha_2} \right] . \tag{3.43}$$

Substituting (3.43) into (3.35) establishes (3.23).

3.3. Reduced-Order Series-Parallel Adaptive Identifier

Consider now the series-parallel (or equation error) identifier configuration [6],[38]. The parallel adjustable model given before is replaced by a series-parallel adjustable discrete system

$$y_s(k) = \sum_{i=1}^{n} \hat{a}_i(k)y(k-i) + \sum_{i=1}^{n} \hat{b}_i(k)u(k-i) = \hat{p}_s^T(k)\theta(k-1) \tag{3.44}$$

$$y_s^o(k) = \hat{p}_s^T(k-1)\theta(k-1) \tag{3.45}$$

where

$$\theta(k-1) = [y(k-1),y(k-2),\ldots,y(k-n),u(k-1),\ldots,u(k-n)]^T \tag{3.46}$$

and

$$\hat{p}_s^T(k) = [\hat{a}_1(k),\hat{a}_2(k),\ldots,\hat{a}_n(k),\hat{b}_1(k),\ldots,\hat{b}_n(k)]. \tag{3.47}$$

The adaption algorithm to be investigated is [6]*

$$\hat{p}_s(k) = \hat{p}_s(k-1) + \frac{F_s(k-1)\theta(k-1)\nu_s^o(k)}{1+\theta^T(k-1)F_s(k-1)\theta(k-1)} \tag{3.48}$$

$$\nu_s^o(k) = y(k) - y_s^o(k) \tag{3.49}$$

where

$$F_s(k) = F_s(k-1) - \frac{F_s(k-1)\theta(k-1)\theta^T(k-1)F_s(k-1)}{\lambda+\theta^T(k-1)F_s(k-1)\theta(k-1)}, \quad \lambda > 0.5. \tag{3.50}$$

<u>Theorem 3.3.1</u>: For the series parallel identifier of (3.44) to (3.49) the composite error

$$Z_s(k) = [y(k)-y_s(k),a_1-\hat{a}_1(k),\ldots,a_n-\hat{a}_n(k),b_1-\hat{b}_1(k),\ldots,b_n-\hat{b}_n(k)]^T \tag{3.51}$$

is bounded for all k and

---

*For $\lambda = 1$ the algorithm (3.48) to (3.50) is the standard least squares algorithm which is not exponentially stable and therefore lags the properties proven in [36].

$$\lim_{k\to\infty} \sup \| Z_s(k) \| \le \mu n \gamma h \| B_f \| \left[\frac{\alpha_1}{1-\mu\alpha_2} \right] \left[\frac{\beta_s c_1}{1-c_2} \right], \tag{3.52}$$

where β_s is related to the bounds on u, y, and F_s and c_1 and c_2 in $c_1(c_2)^{k-k_o}$ bound the k_o to k transition matrix of the linear homogeneous identifier error system (i.e. A_s below).

<u>Proof</u>: The identification error system obtained in Appendix E is

$$Z_s(k+1) = A_s(k) Z_s(k) + B_s(k) y_u(k+1) \tag{3.53}$$

where

$$A_s(k) = \begin{bmatrix} 0 & \theta^T(k) \left\{ I - \dfrac{F_s(k)\theta(k)\theta^T(k)}{1+\theta^T(k)F_s\theta(k)} \right\} \\ 0 & I - \dfrac{F_s(k)\theta(k)\theta^T(k)}{1+\theta^T(k)F_s(k)\theta(k)} \end{bmatrix} \tag{3.54}$$

$$B_s(k) = \begin{bmatrix} \dfrac{1}{1+\theta^T(k)F_s(k)\theta(k)} \\ \dfrac{-F_s(k)\theta(k)}{1+\theta^T(k)F_s(k)\theta(k)} \end{bmatrix}, \tag{3.55}$$

The exponential stability of the homogeneous portion of (3.53) is proven in [36] (with $\lambda = \infty$). Therefore, the bound from [21] used in (3.35) is applicable. However, since in this case $\psi \equiv 0$, the $y_u \equiv 0$ convergence rate of $\| Z_s(k) \|$ is determinable directly from the transition matrix of (3.53) and is not a function of $Z_s(k_o)$ as in the parallel case. Due to the exponential stability of the homogeneous portion of (3.53)

$$\| Z_s(k) \| \le c_1 c_2^{k-k_o} \| Z_s(k_o) \| + \sum_{i=k_o}^{k-1} c_1 c_2^{k-1-i} \| B_s(i) \| \| y_u(i+1) \| . \tag{3.56}$$

From (3.55), due to the boundedness of u, the stability of A_s, and the boundedness of $F_s(k)$, $B_s(k)$ is bounded. Define

$$\beta_s = \sup_i \| B_s(i) \| < \infty. \tag{3.57}$$

Using (3,43) and (3.57) in (3.56) results in (3.52) as $k \to \infty$.

3.4. Discussion and Example

In full-order application parallel and series-parallel identifiers differ in their possible unbiasedness or biasedness, respectively, in the presence of zero-mean output measurement noise and in the presence or absence of a priori plant information requirements, respectively, as indicated by the strict positive reality requirement in (3.21) for parallel identifiers and the lack of any such requirement for series-parallel identifiers. As shown in this study these two classes also demonstrate differences in reduced-order application since the bound (3.23) for parallel identifiers is dependent on the initial composite error and the "homogeneous" algorithm convergence rate but for series-parallel identifiers on the identifier error system input matrix norm maximum and the "homogeneous" algorithm convergence rate but <u>not</u> the initial composite error. However both bounds share the common factor

$$\mu \gamma n g \tag{3.58}$$

where $g = h \| B_f \| \dfrac{\alpha_1}{1 - \mu\alpha_2}$ showing that both identifiers are robust with respect to unmodeled parasitics in the sense that as $\mu \to 0$ the error bounds go to zero. The dependence of (3.58) on $\gamma = \sup_i |u(i+1) - u(i)|$ is an indicator of the influence of the input characteristics on the identification results. The bounds indicate possibilities for reducing the error by a proper choice

of the input signal. That is the identification results in the presence of parasitics can be improved by choosing an input signal which is sufficiently rich but has a low $\gamma = \sup_i |u(i+1) - u(i)|$. Some of the terms in the bounds obtained are difficult or even impossible to calculate since they depend on the characteristics of the unknown parasitics. Thus the bound can only be used for qualitative analysis and not as a practical measure for evaluating identification results in any given practical situation. Such a qualitative analysis indicates possible improvement of the identification results by a proper choice of the input signal and an increasing rate of convergence of the algorithm.

The effect of μ, the input characteristics γ and the initial composite error on the identification results has been demonstrated by simulating the following example:

$$\begin{bmatrix} x(k+1) \\ \hline z(k+1) \end{bmatrix} = \left[\begin{array}{cc|cc} -0.2 & 1 & \mu & 0 \\ 0.48 & 0 & 0 & 0.5\mu \\ \hline 0 & 0 & 0.5\mu & 0 \\ 0 & 0 & 0 & 0.7\mu \end{array}\right] \begin{bmatrix} x(k) \\ \hline z(k) \end{bmatrix} + \left[\begin{array}{c} 1 \\ 0 \\ \hline 0.8 \\ 0.1 \end{array}\right] u(k) \tag{3.59}$$

$$y(k) = [1 \quad 0]\, x(k) \tag{3.60}$$

which can be expressed in the form of (3.12), (3.13) by using the transformation (3.4) i.e.

$$y(k) = -0.2y(k-1) + 0.48y(k-2) + \left(1 + \frac{0.8\mu}{1-0.5\mu}\right)u(k-1)$$

$$+ \left(\frac{0.4\mu}{1-0.5\mu} + \frac{0.1\mu}{1-0.7\mu}\right)u(k-2) + y_u(k)\,, \tag{3.61}$$

$$y_u(k) = \mu(\eta_1(k-1) + 0.5\eta_1(k-2) + \eta_2(k-1)) \tag{3.62}$$

The criteria used for evaluation of the identification results are the parametric distance defined by $D = [\sum_{i=1}^{n}(a_i - \hat{a}_i(k))^2 + (b_i - \hat{b}_i(k))^2]^{1/2}$ and the output error $e = y(k) - \hat{y}(k)$. For the input excitation signal we have chosen a sequence of pulses whose frequency and amplitude have been adjusted to give a particular value of γ.

The effect of the initial error on the performance of the parallel identifier is shown in Fig. 3.2 and Fig. 3.3. That is by increasing the norm of the initial error from 1 to 15 the "convergent" parametric distance increases by 55.2% (i.e. from 0.087 to 0.135). The same change in the initial error has no effect on the performance of the series-parallel identifier as indicated by Fig. 3.4 and Fig. 3.5. The effect of μ on the identification results is shown in Figs. 3.4 and 3.6. By increasing μ by 5 times, the "convergent" parametric distance increases by 5.44 times and the oscillations of the output error become higher. This shows that when the parasitics are sufficiently fast the identification results can be satisfactory. The effect of the characteristics of the input on the identification results is demonstrated by simulations and is shown in Figs. 3.4 and 3.7. By increasing $\gamma = \sup_i |u(i+1)-u(i)|$ from 13 to 100 the "convergent" parametric distance increases by 5.4% and the output error deviations from zero increase considerably.

For both classes of algorithms the "homogeneous" i.e., in the absence of parasitics, convergence rate is a complicated function of input $\{u(k)\}$ composition and the step-size matrix $F(k)$. For parallel identifiers the error smoothing coefficients c_i in (3.21) also impact the convergence rate. Clearly, further refinement of reduced-order usage composite error bounds will require more detailed descriptions of the convergence rate mechanics, which is an admittedly [21] difficult task.

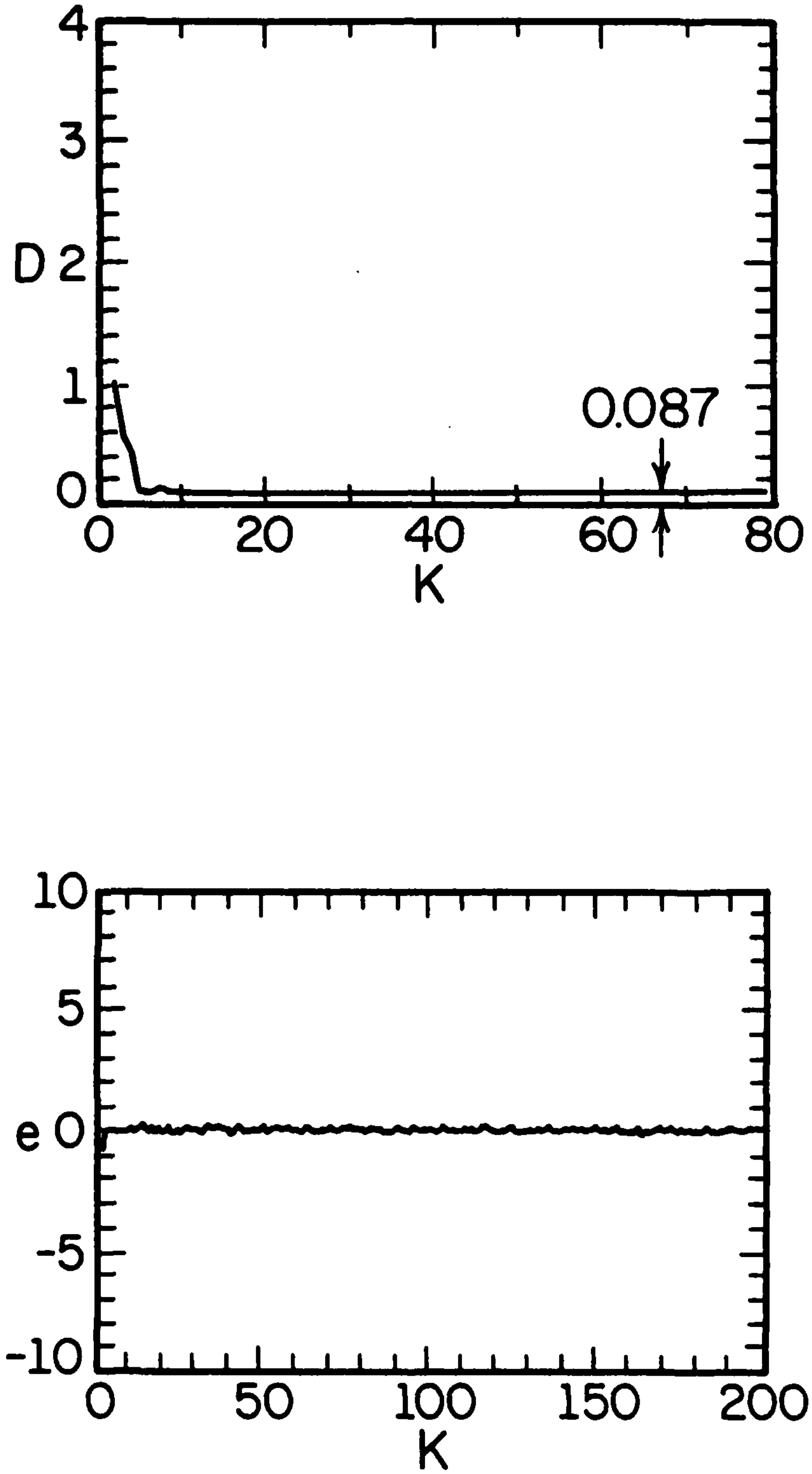
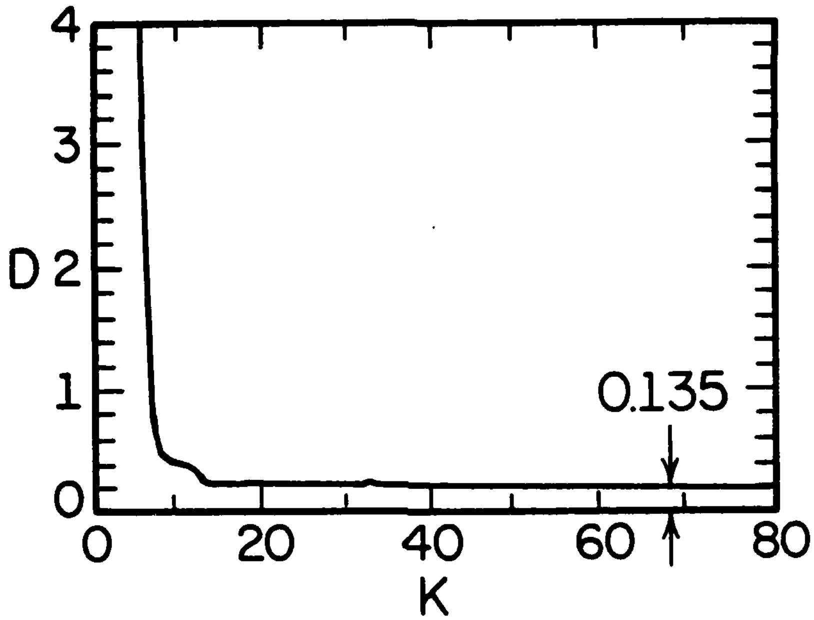


Fig. 3.2. Identification results using the parallel identifier for $\mu = 0.1$, $\gamma = 13$, and $\|Z(Ko)\| = 1$.



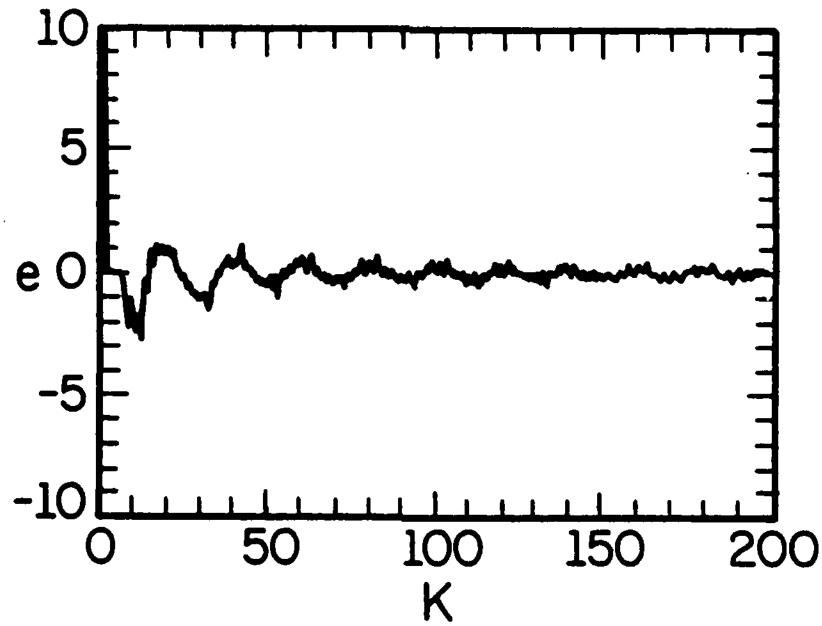


Fig. 3.3 Identification results using the Parallel identifier for $\mu = 0.1$, $\gamma = 13$ and $||Z(Ko)|| = 15$.

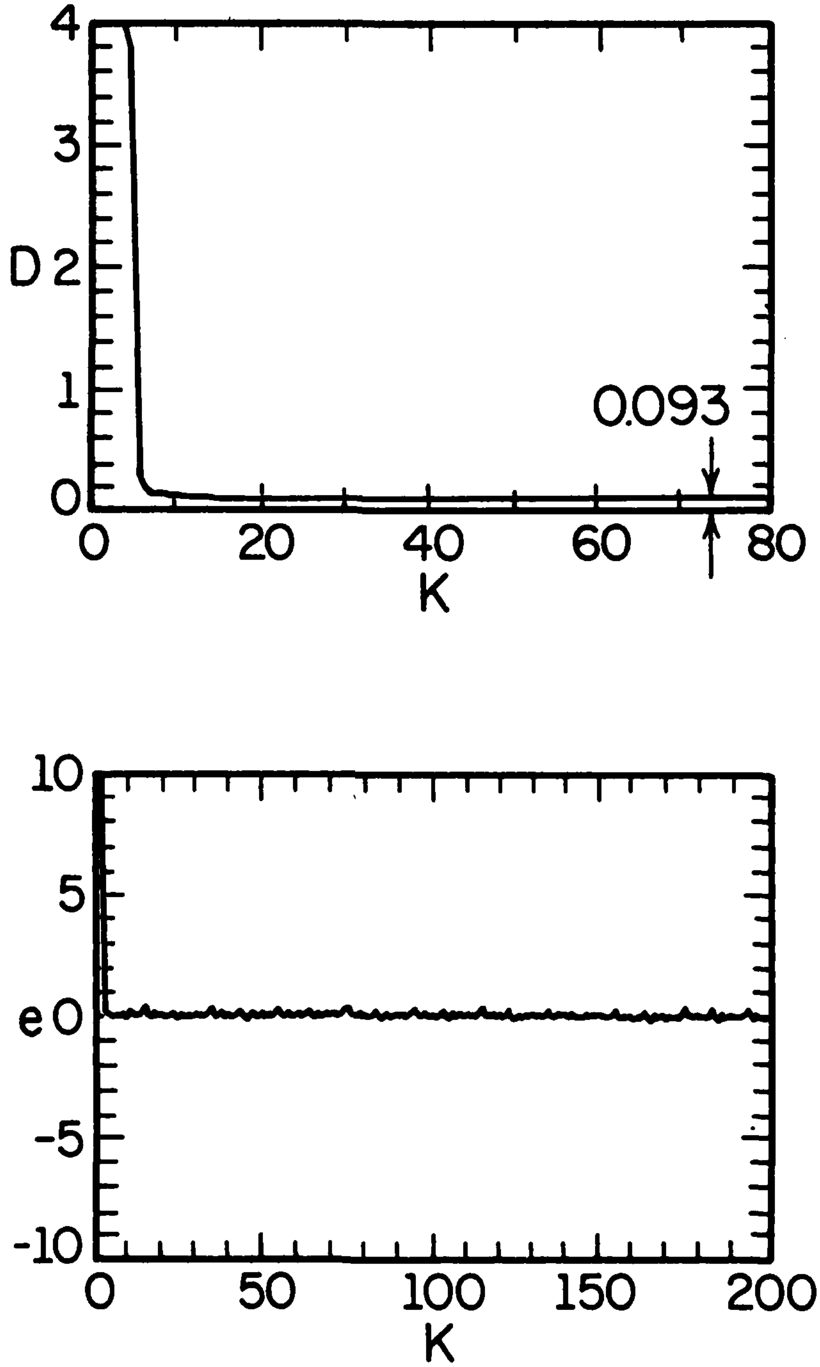


Fig. 3.4 Identification results using the Series-Parallel identifier for μ = 0.1, γ = 13 and $||Z(ko)||$ = 1.

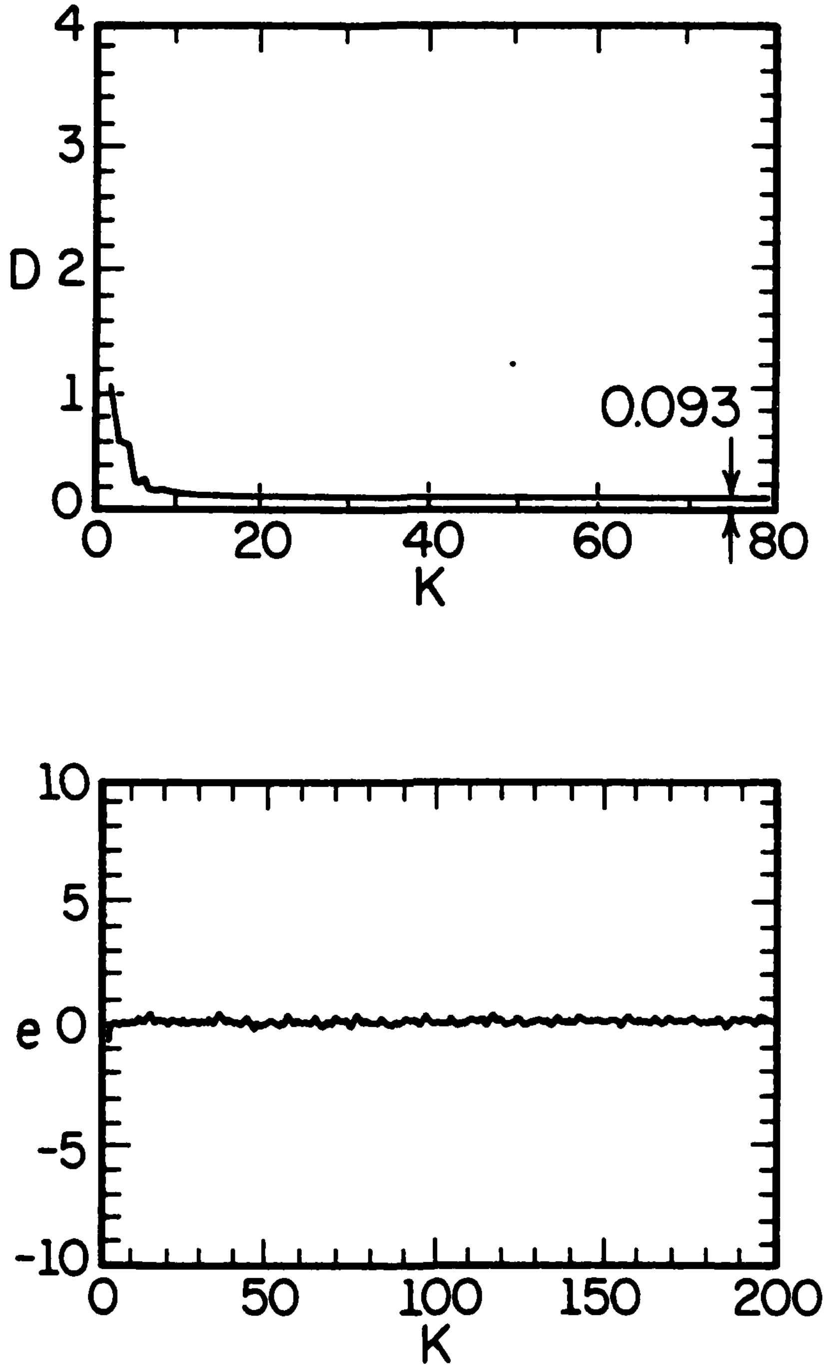
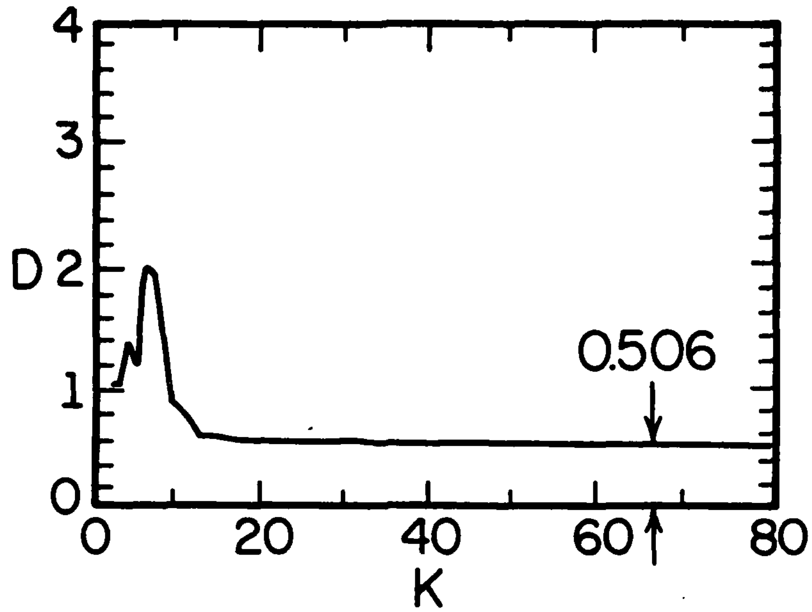


Fig. 3.5 Identification results using the Series-Parallel identifier for $\mu = 0.1$, $\gamma = 13$ and $\|Z(Ko)\| = 15$.



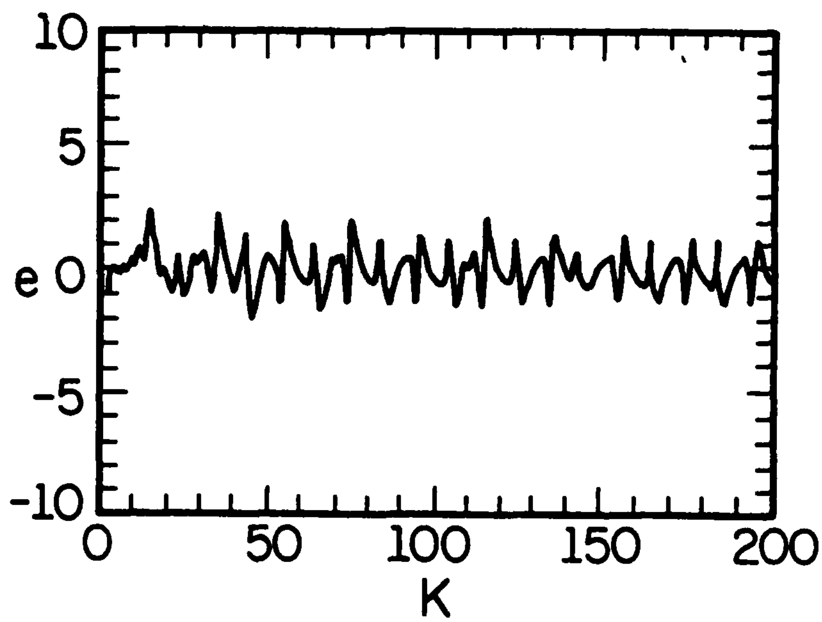


Fig. 3.6 Identification results using the Series-Parallel identifier for $\mu = 0.5$, $\gamma = 13$ and $\|Z(Ko)\| = 1$.

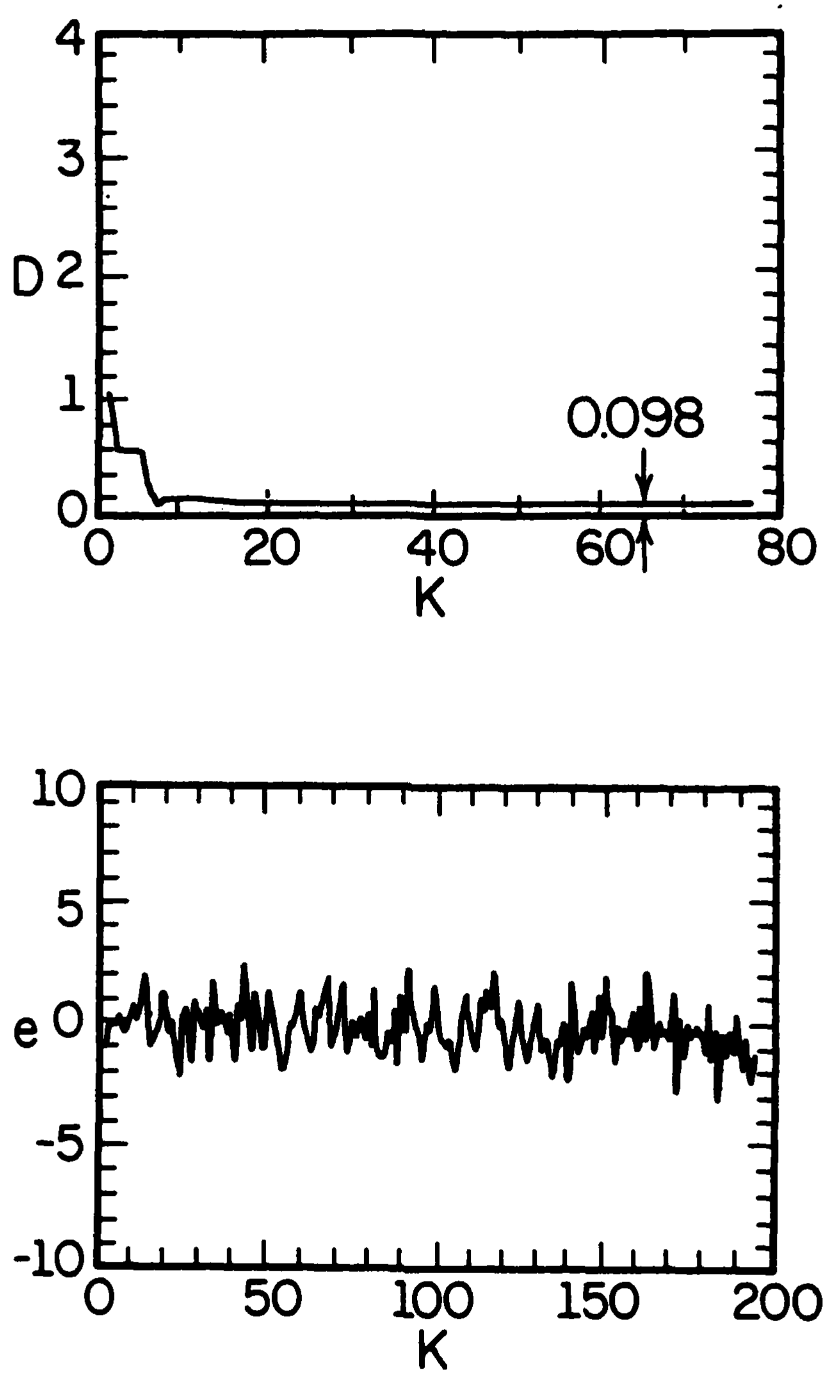


Fig. 3.7 Identification results using the Series-Parallel identifier for $\mu = 0.1$, $\gamma = 100$ and $\|Z(Ko)\| = 1$.

4. ROBUSTNESS IMPROVEMENT OF IDENTIFIERS AND ADAPTIVE OBSERVERS

4.1. Introduction

In Sections 2 and 3 it was shown that several adaptive schemes are robust with respect to modeling errors consisting of fast parasitics which are weakly observable in the plant output. In this section we show that the assumption of weak observability is crucial for robustness. If the parasitic modes are strongly observable the schemes are no longer robust. That is the error due to model-plant mismatch can be large even when the fast modes are infinitely fast. In the case of continuous-time adaptive schemes we show that the addition of a low pass filter at the output makes the parasitics weakly observable and hence guarantees the robustness of the enlarged scheme. The robustness of discrete-time adaptive schemes with respect to strongly observable parasitics can be guaranteed by modifying the adjustable model and the adaptive laws.

4.2. A Model with Fast Parasitics

A SISO system possessing slow and strongly observable fast parts can be represented in the explicit singular perturbation form [25]

$$\dot{x} = A_{11}x + A_{12}x_f + b_1u \tag{4.1}$$

$$\mu\dot{x}_f = A_{21}x + A_{22}x_f + b_2u \tag{4.2}$$

$$y = C_1x + C_2x_f \tag{4.3}$$

where $x \in R^n$; $x_f \in R^m$ and μ is a small positive parameter associated with the presence of "parasitic" elements, such as the time constants, masses, etc.

Without altering the input-output characteristics of the system we use the transformation

$$\eta = x_f + Lx + A_f^{-1}\bar{b}_f u \qquad (4.4)$$

to obtain

$$\dot{x} = A_o x + b_o u + A_{12}\eta \qquad (4.5)$$

$$\mu\dot{\eta} = A_f\eta + \mu b_f\dot{u} \qquad (4.6)$$

$$y = C_o x + C_2\eta - C_2 b_f u \qquad (4.7)$$

where $A_o = A_{11} - A_{12}L$, $A_f = A_{22} + LA_{12}$, $\bar{b}_f = b_2 + Lb_1$, $b_o = b_1 - A_{12}A_f^{-1}\bar{b}_f$, $C_o = C_1 - C_2L$, $b_f = A_f^{-1}\bar{b}_f$, and L satisfies the algebraic equation [26]

$$A_{22}L - A_{21} + \mu LA_{12}L - \mu LA_{11} = 0 \qquad (4.8)$$

Assuming that the pair (C_o, A_o) is completely observable (4.5) to (4.7) can be written in the following "modal" canonical form

$$\dot{x} = \left[\begin{array}{c|c} -a_o & h^T \\ \hline & \Lambda \end{array}\right] x + bu + (C_3A_f + \mu A_1)\sigma \qquad (4.9)$$

$$\mu\dot{\sigma} = A_f\sigma + b_f\dot{u} \qquad (4.10)$$

$$Y = CX = y, \qquad C = [1,0,\ldots,0] \qquad (4.11)$$

where $A_1 = A_{12} - a_oC_2$, $b = b_o + a_oC_2b_f$, $h^T = [1\ 1\ \ldots\ 1]$, $C_3 = C^TC_2$, Λ is an $(n-1)\times(n-1)$ diagonal matrix with arbitrary but known constant and negative diagonal elements $-\lambda_i$ $(i=2,\ldots,n)$. Here the fast state σ is defined by*

*Note that for the initial conditions of σ to be finite as $\mu\to 0$ the initial conditions of η in (4.12) have to be at most of $O(\mu)$.

$$\eta = \mu\sigma. \tag{4.12}$$

The reduced order system obtained by setting $\mu=0$ in (4.9) to (4.11) is

$$\dot{\bar{X}} = \left[-\bar{a}_o \begin{array}{|c} h^T \\ \hline \Lambda \end{array}\right] \bar{X} + \bar{b}u - \left[\begin{array}{c} 1 \\ \hline 0 \end{array}\right] C_2 \bar{b}_f \dot{u} \tag{4.13}$$

$$\bar{Y} = C\bar{X} = \bar{y}. \tag{4.14}$$

We point out that in Section 2 we assumed $C_2 = 0$, therefore the parasitics were weakly observable. Here we allow $C_2 \neq 0$. This means that as $\mu \to 0$ the transfer function of the system becomes proper. Thus the input dependent signal $-C_2 b_f \dot{u}$ is present in the dominant part of the plant and remains in the reduced order system. Since the continuous-time adaptive schemes are designed for strictly proper systems the appearance of $-C_2 b_f \dot{u}$ cannot be seen by the adaptive algorithm. The purpose of this section is to investigate the behavior of an n-th order observer with respect to parasitics when $C_2 \neq 0$ and compare with that in the case of $C_2 = 0$. We will first show that the appearance of the input $-C_2 b_f \dot{u}$ in the reduced order system is equivalent to the nonrobustness of the adaptive observer with respect to parasitics. Then we will show how the robustness of the adaptive scheme can be reestablished using a low-pass filter at the output of the plant. The following assumptions are made:

(A1) A_o and A_f are stable.

(A2) The order n of the dominant part of the plant is known.

(A3) The triple (A_o, b, C) is completely controllable and observable.

(A4) The only available signals are $u(t)$ and $y(t)$.

(A5) $u(t)$ and $\dot{u}(t)$ are piecewise continuous bounded functions of time.

4.3. Minimal Form Adaptive Observer

Suppose that an n-th order adaptive observer is designed to estimate the state vector X of the dominant part of the plant and identify the unknown vectors a_o and b. It is assumed that the dominant part of the system is strictly proper and the presence of parasitics is disregarded during the design. The stability of the adaptive observer when applied to the real plant with a proper dominant part and parasitics is analyzed.

The algorithm [32],[33] for the n-th order adaptive observer based on the system (4.13),(4.14) without the signal $-C_2\bar{b}_f\dot{u}$ is given by the equations (4.15) through (4.16) below. The observer equation is

$$\dot{\hat{X}} = K\hat{X} + [k-\hat{a}(t)]y + \hat{b}(t)u + w + r \tag{4.15}$$

$$\hat{y} = \hat{X}_1 \tag{4.16}$$

where w and r are auxiliary signals formed by the output error $e_1 \triangleq \hat{y}-y$, the derivatives of the parameter error components and the components

$$v_1 = y, \quad q_1 = u, \quad v_i = \frac{1}{s+\lambda_i}\, y, \quad q_i = \frac{1}{s+\lambda_i}\, u \qquad (i=2,\ldots,u) \tag{4.17}$$

of the vectors v and q as follows

$$w = \begin{bmatrix} 0 \\ \dot{\phi}_2 v \\ \dot{\phi}_n v \end{bmatrix}, \qquad r = \begin{bmatrix} 0 \\ \dot{\Psi}_2 q \\ \dot{\Psi}_n q \end{bmatrix}. \tag{4.18}$$

Moreover $K = \left[\begin{array}{c|c} -\lambda_1 & h^T \\ \hline 0 & \\ \vdots & \Lambda \\ 0 & \end{array}\right]$ is stable, the transfer function $C^T(sI-K)^{-1}d = \frac{1}{s+\lambda_1}$ is strictly positive real, and $d = (1\ 0\ \ldots\ 0)^T$. The adaptive laws for adjusting the parameters are

$$\dot{\phi} = -\Gamma e_1 v \tag{4.19}$$

$$\dot{\Psi} = -Me_1 q \tag{4.20}$$

where $\phi = \hat{a}(t) - a_o$ and $\Psi = \hat{b}(t) - b$.

It can be shown as in Section 2 that the behavior of the adaptive observer in the presence of parasitics is equivalent to the stability of the following set of differential equations

$$\dot{\varepsilon} = K\varepsilon + d[\phi^T v + \Psi^T q] - (C_3 A_f + \mu A_1)\sigma \tag{4.21}$$

$$\varepsilon_1 = e_1 \tag{4.22}$$

$$\dot{\phi} = -\Gamma \varepsilon_1 v \tag{4.23}$$

$$\dot{\Psi} = -M\varepsilon_1 q \tag{4.24}$$

which using the composite error $Z(t) = [\varepsilon^T, \phi^T, \Psi^T]^T$ become

$$\dot{Z}(t) = A_\eta(t) Z(t) + H\sigma(t) \tag{4.25}$$

where

$$A_\eta(t) = \left[\begin{array}{c|c|c} K & dv^T & dq^T \\ \hline -\Gamma v & & \\ \cline{1-1} & 0 & 0 \\ -Mq & & \end{array}\right], \qquad H = -\left[\begin{array}{c} C_3 A_f + \mu A_1 \\ \hline 0 \end{array}\right]. \tag{4.26}$$

The following theorem establishes the boundedness of the composite error $Z(t)$ in the presence of parasitics.

<u>Theorem 4.3.1</u>: If $u(t)$ is dominantly rich for the plant (4.9) to (4.11) then the homogeneous part of (4.25) is u.a.s. and the error vector $Z(t)$ is bounded by

$$\lim_{t\to\infty} \sup \|Z(t)\| \le (\|C_3 A_f\| + \mu\|A_{12} - a_o C_2\|) \frac{m_1}{m_2} \frac{\alpha_1}{\alpha_2} \|b_f\| \tag{4.27}$$

where m_1, m_2, α_1, and α_2 are positive constants in the bounds $\|\phi_\eta(t,\tau)\| \le$

$\leq m_1 e^{-m_2(t-\tau)}$, $\|\phi_f(t,\tau)\| \leq \alpha_1 e^{-\alpha_2(\frac{t-\tau}{\mu})}$, $\phi_\eta(t,\tau)$, $\phi_f(t,\tau)$ are the transition matrices of (4.25) and (4.10) respectively, and $\gamma = \sup_t |\dot{u}(t)|$.

Proof: The proof of Theorem 4.1.1 follows directly from Lemma 2.1.1 and Theorem 2.3.1 of Section 2 and is not repeated here.

When C_2=0 the algorithm is robust with respect to parasitics because from bound (4.27) the identification and observation error goes to zero as $\mu \to 0$. However, when $C_2 \neq 0$ the algorithm is no longer robust with respect to parasitics. That is, as $\mu \to 0$ (i.e. modeling error $\to$ 0) the composite error, although bounded, does not go to zero. We show this by applying the adaptive observer to the reduced order system (4.13),(4.14). In this case the stability properties of the adaptive observer are equivalent to the stability of the following set of error equations

$$\dot{\bar{\varepsilon}} = K\bar{\varepsilon} + d[\bar{\phi}^T v + \bar{\Psi}^T q] + C_3 \bar{b}_f \dot{u} \tag{4.26}$$

$$\bar{\varepsilon}_1 = \bar{e}_1 = \hat{y} - \bar{y} \tag{4.27}$$

$$\dot{\bar{\phi}} = -\Gamma \bar{\varepsilon}_1 v \tag{4.28}$$

$$\dot{\bar{\Psi}} = -M \bar{\varepsilon}_1 q \tag{4.29}$$

which in compact form is

$$\dot{\bar{Z}}(t) = A_\eta(t)\bar{Z}(t) + \bar{H} C_3 b_f \dot{u} \tag{4.30}$$

where $\bar{Z}(t) = [\bar{\varepsilon}^T, \bar{\phi}^T, \bar{\Psi}^T]^T$, and $\bar{H} = \begin{bmatrix} C^T \\ \hline 0 \end{bmatrix}$. The homogeneous part of (4.30) is uniformly asymptotically stable for u sufficiently rich. Thus $\bar{Z}(t)$ is bounded but not zero since $\dot{u}$ is a bounded general input signal. The loss of robustness is due to the fact that the adaptive observer is designed for strictly proper systems and therefore the signal $C_3 \bar{b}_f \dot{u}$ in the proper system (4.13),(4.14) cannot be seen by the algorithm.

In the following section we show that the robustness of the adaptive observer can be established by using a first order low pass filter at the output of the plant.

4.4. Filter Design

The strong observability of the fast parasitic modes is the cause for the loss of robustness of the adaptive observer. We will show that this can be avoided at the cost of increasing the order of the adaptive observer and adaptive laws by one. Consider the following first order filter for filtering the output of the unknown plant

$$\dot{w} = -a_s w + d_s y \qquad , \; a_s > 0 \tag{4.31}$$

$$y_s = w. \tag{4.32}$$

Then (4.31), (4.32) augmented with (4.5)-(4.7) becomes

$$\dot{z}_o = \tilde{A} z_o + \tilde{b} u + \tilde{H}\eta \tag{4.33}$$

$$\mu\dot{\eta} = A_f \eta + \mu b_f \dot{u} \tag{4.34}$$

$$y_s = [1 \quad 0 \quad \dots \quad 0] z_o = w \tag{4.35}$$

where $\tilde{A} = \left[\begin{array}{c|c} d_s & d_s C_o \\ \hline 0 & A_o \end{array}\right]$, $\tilde{b} = \left[\begin{array}{c} -d_s C_2 b_f \\ \hline b_o \end{array}\right]$, $\tilde{H} = \left[\begin{array}{c} C_2 d_s \\ \hline A_{12} \end{array}\right]$ and $z_o = (w, x^T)^T$. The only restrictions on a_f, d_f is that controllability and observability of the augmented system should be preserved. Thus without loss of generality we can express (4.33), (4.35) in the "modal" canonical form

$$\dot{z} = \left[\begin{array}{c|c} -a_s & h^T \\ \cline{2-2} & \Lambda \end{array}\right] z + b_s u + H\eta \tag{4.36}$$

$$\mu\dot{\eta} = A_f\eta + \mu b_f\dot{u} \tag{4.37}$$

$$y_s = [1\ 0\ \dots\ 0]z = w. \tag{4.38}$$

The order of (4.36) is n+1 and Λ and h^T are defined appropriately as in (4.9). The (n+1)-th adaptive observer for estimating z and identifying a_s and b_s under the assumption that $\eta=0$ is described by (4.15) through (4.20) where y is replaced by y_s and the signals in the algorithm are of order n+1.

The stability of the (n+1)-th order adaptive observer in the presence of parasitics is equivalent to the stability of the following set of differential equations

$$\dot{\varepsilon} = K\varepsilon + d[\phi^T v + \Psi^T q] - H\eta \tag{4.39}$$

$$\varepsilon_1 = e_1 = \hat{y} - y_s \tag{4.40}$$

$$\dot{\phi} = -\Gamma\varepsilon_1 v \tag{4.41}$$

$$\dot{\Psi} = -M\varepsilon_1 q \tag{4.42}$$

which in compact form become

$$\dot{Z}(t) = A_\eta(t)Z(t) + H_\eta\eta \tag{4.43}$$

where $Z(t)$ and $A_\eta(t)$ are defined appropriately as in (4.25) and $H_\eta = \begin{bmatrix} -H \\ --- \\ 0 \end{bmatrix}$. The following theorem establishes the robustness of the (n+1)-th order adaptive observer with respect to parasitics and gives a bound for the composite error.

Theorem 4.4.1: If u(t) is dominantly rich for the plant (4.36) to (4.38), then the homogeneous part of (4.43) is uniformly asymptotically stable and the composite error vector Z(t) is bounded by

$$\lim_{t\to\infty} \sup \|Z(t)\| \le \mu\gamma \frac{n_1}{n_2}\frac{\alpha_1}{\alpha_2}\|H\|\|b_f\| \tag{4.44}$$

where n_1, n_2 are positive constants in the bound $\|\phi(t,\tau)\| \le n_1\exp(-n_2(t-\tau))$ for

the transition matrix $\phi(t,\tau)$ of (4.42). The constants γ, α_1, and α_2 are as defined in Theorem 4.3.1.

Proof: The proof follows directly from Lemma 2.1.1 and Theorem 2.3.1 of Section 2 by noticing the similarity of (4.43) with (2.13).

4.5. Discrete-Time Identifier

Consider system (3.1), (3.2) of Section 3 but with an output of the form

$$y(k) = C_1 x(k) + C_2 z(k) \tag{4.45}$$

instead of (3.3). By using the transformation

$$\eta(k) = Z(k) + Px(k) - B_f u(k) + \bar{B}_f(u(k)-u(k-1)) \tag{4.46}$$

and assuming that the dominant part is in the observable canonical form we obtain the following representation for the plant.

$$x_s(k+1) = A_x x_s(k) + (aC_2 + \mu H)\eta(k) + b_s u(k) + (aC_2 + \mu A_{12})B_f u(k-1) \tag{4.47}$$

$$\eta(k+1) = \mu A_f \ (k) + \mu A_f B_f[(u(k+1)-u(k))-(u(k)-u(k-1))] \tag{4.48}$$

$$y(k) = [1 \ \ 0 \ \ldots \ 0]x_s(k) \tag{4.49}$$

where

$$A_s = \left[\begin{array}{c|c} -a & I \\ \hline & 0 \end{array}\right], \qquad H = A_{12} - [A_f^T C_2^T \ \vdots \ 0]^T$$

$$b_s = b - \mu a C_2 A_f B_f + [B_f^T (I-2\mu A_f)^T C_2^T \ \vdots \ 0]^T$$

$$B_f = (I-\mu A_f)^{-1}(B_2 + PB_1), \quad \bar{B}_f = (I-\mu A_f)^{-1} B_f \tag{4.50}$$

and P satisfies (3.8).

Due to this observable canonical form we can write (4.47) to (4.49) as

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n+1} b_i u(k-i) + y_v(k) \tag{4.51}$$

where

$$y_v(k) = \sum_{i=1}^{n} g_i \eta(k-i) \tag{4.52}$$

and g_i is the ith row of $aC_2 + \mu H$.

The reduced-order system obtained by setting $\mu=0$ in (4.51), (4.52) is

$$\bar{y}(k) = \sum_{i=1}^{n} \bar{a}_i \bar{y}(k-i) + \sum_{i=1}^{n+1} \bar{b}_i u(k-i). \tag{4.53}$$

We point out that in Section 2 we assume $y(k) = C_1 x(k) + \mu C_2 z(k)$, therefore the parasitics were weakly observable. Here we allow $y(k) = C_1 x(k) + C_2 z(k)$. This means that as $\mu \to 0$ a throughput is introduced in the output which causes an extra delayed input term $b_{n+1}u(k-n-1)$ to appear in both the dominant part of the plant and the reduced order system. The adaptive identifiers described by (3.14) to (3.21) and (3.44) to (3.50) cannot see the term $b_{n+1}u(k-n-1)$. Thus if we apply the nth order identifier described by (3.14) to (3.21) or (3.44) to (3.50) to the plant (4.51), (4.52) we will obtain an error system equation of the form (3.26) or (3.53) but with an additional disturbance input due to the unmodeled term $b_{n+1}u(k-n-1)$. This disturbance causes an output/parameter error which is non-zero even when the parasitics are infinitely fast, i.e. $\mu=0$ and therefore the scheme is no longer robust. The robustness of discrete-time identifiers with respect to strongly observable parasitics can be established by modifying the identifiers to take into account the term $b_{n+1}u(k-n-1)$ introduced by the parasitics. For the nth-order

parallel identifier described by (3.14) to (3.21), we introduce the following modification: The estimation or adjustable model is modified to

$$y_p(k) = \sum_{i=1}^{n} \hat{a}_i(k) y_p(k-i) + \sum_{i=1}^{n+1} \hat{b}_i(k) u(k-i) = \hat{p}_o(k)\theta_o(k-1) \tag{4.54}$$

$$y_p^o(k) = \hat{p}_o^T(k-1)\theta_o(k-1) \tag{4.55}$$

where

$$\hat{p}_o^T(k) = [\hat{a}_1(k) \ \ldots \ \hat{a}_n(k) \ \hat{b}_1(k) \ \ldots \ \hat{b}_n(k), \hat{b}_{n+1}(k)],$$

$$\theta_o(k-1) = [y_p(k-1) \ldots y_p(k-n) u(k-1) \ldots u(k-n), u(k-n-1)]^T. \tag{4.56}$$

The adaptation laws for updating $\hat{p}_o(k)$ have the same form as (3.18) to (3.20) but $\hat{p}_p(k)$ and $\theta_p(k)$ are replaced with $\hat{p}_o(k)$ and $\theta_o(k)$ respectively. The dimension of the adjustable gain $F_p(k)$ is defined appropriately and the stability condition (3.21) remains the same. A similar modification can be introduced for the series-parallel identifier (3.44) to (3.50). Applying the modified nth order adaptive identifiers to the plant (4.51), (4.52) with strongly observable parasitics and following the procedure of Section 3, we can show that the scheme is robust, that is, the composite output/parameter error is of $O(\mu)$.

4.6. Discussion and Example

The loss of robustness in the case of strongly observable parasitics is due to the fact that the parasitics introduce a throughput in the reduced-order (μ=0) system. Continuous-time adaptive schemes are designed for strictly proper systems and are no longer uniformly asymptotically stable when applied to proper systems. In the case of discrete-time identifiers

the parasitics introduce an extra delayed input in the ARMA model which cannot be seen by the identifier. This input acts as a persistent disturbance in the composite error equation for all μ. By modifying the adaptive schemes at the expense of increasing their order by one we re-establish robustness with respect to strongly observable parasitics.

We demonstrate the effect of strongly observable parasitics on the continuous-time adaptive observer by digital simulation for the plant,

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix} x_f + \begin{bmatrix} 1.45 \\ 2.25 \end{bmatrix} u \tag{4.57}$$

$$\mu \dot{x}_f = -4x_f - 2u \tag{4.58}$$

$$y = [1 \quad 0]x + x_f \tag{4.59}$$

which is transformed into

$$\begin{bmatrix} \dot{y} \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} \begin{bmatrix} y \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.75 \\ -3 \end{bmatrix} u + \begin{bmatrix} \mu 5.9-4 \\ \mu 10.5 \end{bmatrix} \sigma \tag{4.60}$$

$$\mu \dot{\sigma} = -4\sigma + 0.5\dot{u} \tag{4.61}$$

$$Y = y. \tag{4.62}$$

The vectors to be identified are $a = [5, 10]^T$ and $b = [0.75, -3]^T$. The observation and identification results are summarized in Figs. 4.1 and 4.2. Figures 4.1a,b,c show that although the parasitics are infinitely fast (i.e. $\mu=0$) the identification and observation error is significant. When modeling error is present ($\mu=0.1$) Figs. 4.2a,b,c show that the identification and observation errors are more oscillatory. In both cases ($\mu=0$, 0.1) the identification results are erroneous and the observation error considerable. The effect of weakly

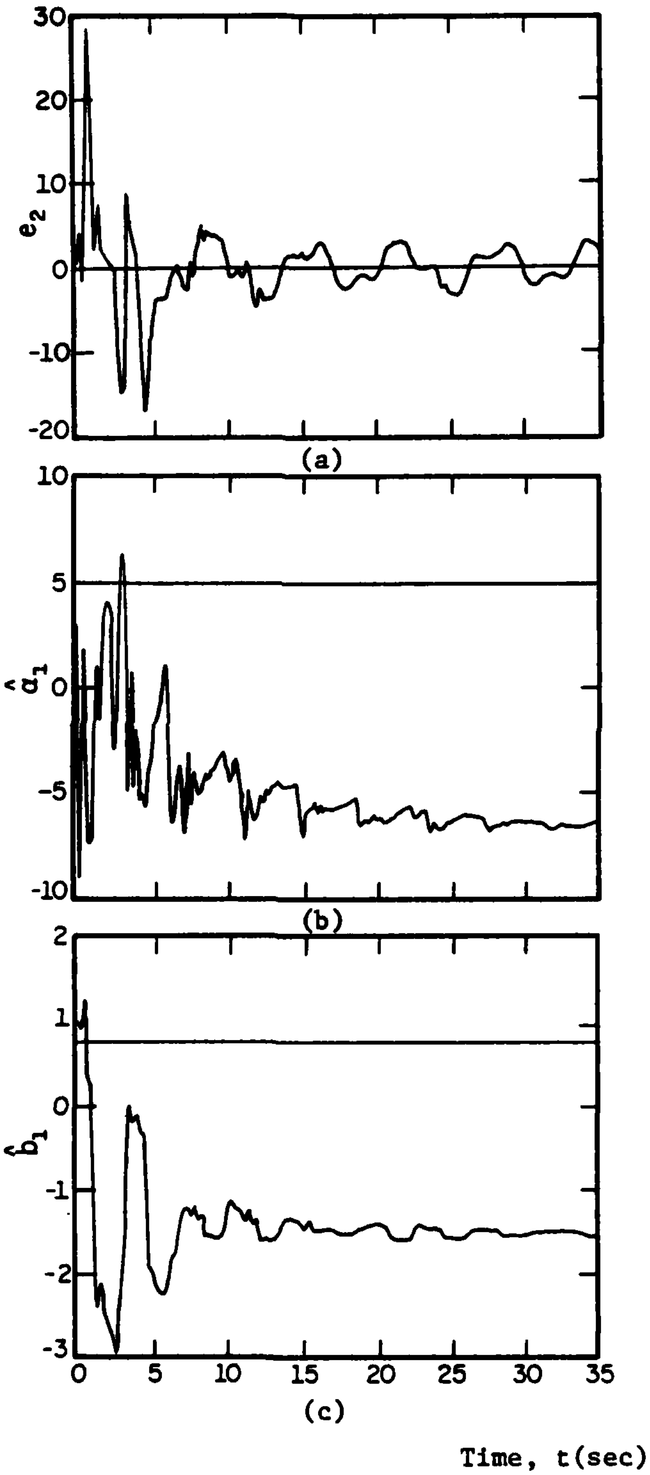


Fig. 4.3 Identification results for μ = 0.0.

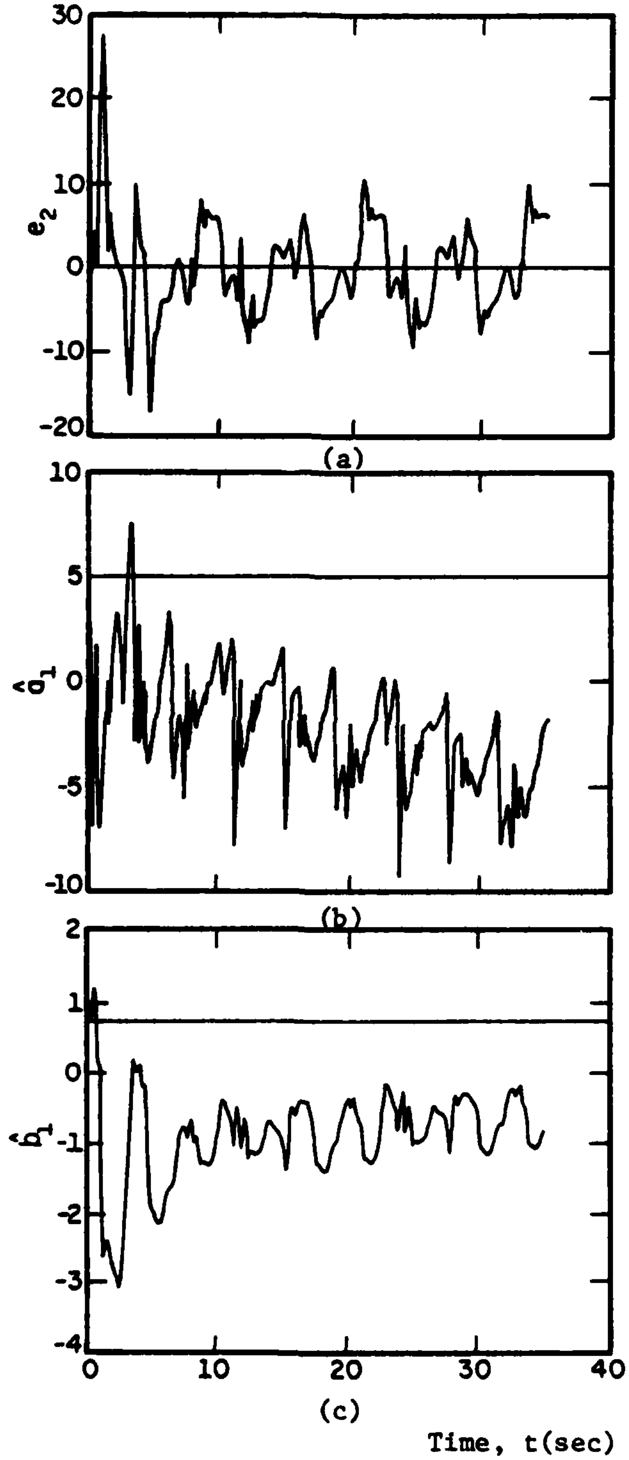


Fig. 4.4 Identification results for μ = 0.1.

observable parasitics on the identification and observation results has been simulated in Section 2 where it is shown that the identification error is almost linear with μ and the adaptive results are exact when there is no modeling error ($\mu=0$). Thus the introduction of the low-pass filter at the output makes the parasitics weakly observable and gives satisfactory identification results provided the parasitics are sufficiently fast as it was emphasized in Section 2. The results obtained in this section can be extended to all other continuous and discrete-time identifiers and adaptive observers.

5. REDUCED-ORDER ADAPTIVE CONTROL

5.1. Introduction

Several attempts have been made to formulate and analyze reduced-order adaptive control schemes. In [15] local stability has been proved for a reduced-order indirect adaptive regulator. Efforts on reduced-order direct adaptive control [22-24] have been restricted to single first or second order examples rather than the general problem. In these examples it was shown by simulations [22],[23], or "linearization" [24] that unmodeled parasitics can lead to instability. The analysis in Section 2 of the effects of high frequency plant inputs on the performance of identifiers and adaptive observers with parasitics has determined that the inputs should be restricted to dominantly rich inputs. As a design concept the dominant richness requires that in the presence of parasitics the richness condition be satisfied outside the parasitic range. It excludes wideband inputs such as noise and square waves as undesirable. The situation in adaptive control is more difficult because the plant input is generated by adaptive feedback which incorporates the unknown plant with parasitics. The schemes so far do not contain a mechanism to restrict the frequency content of the plant input. The lack of this mechanism has caused the loss of robustness reported in [22-24].

The two main results of this section are, first, an estimate of the region of attraction for adaptive regulation and second, a modification of the adaptive laws to guarantee boundedness in the case of tracking. The frequency content and magnitude of the reference input signal, the speed ratio μ of slow versus fast phenomena, the adaptive gain and initial conditions are shown to have crucial effects on the stability of the adaptive control schemes. These results are first analytical conditions for robustness of direct

adaptive control with respect to high frequency dynamics. They are obtained for a continuous-time SISO adaptive control scheme [7]. The same methodology can be extended to more complicated continuous and discrete-time adaptive control problems. The section is organized in two main subsections. The first contains a simple motivating scalar example which illustrates the salient features of the general methodology developed in the second subsection.

5.2. The Scalar Reduced-Order Adaptive Control Problem

We start with a simple example of reduced-order adaptive control in which the output y_p of a second order plant

$$\dot{y}_p = a_p y_p + 2z - u, \qquad a_p > 0 \tag{5.1}$$

$$\mu\dot{z} = -z + u \tag{5.2}$$

with unknown constant parameters a_p and μ, is required to track the state y_m of a first order model

$$\dot{y}_m = -a_m y_m + r(t) \qquad a_m > 0 \tag{5.3}$$

where u is the control input and $r = r(t)$ is a reference input, a uniformly bounded function of time. This example serves as a motivation for and an introduction to the general methodology to be developed in the next subsection. As in Section 2, the model-plant mismatch is due to some "parasitic" time constants which appear as multiples of a singular perturbation parameter μ and introduce the "parasitic" state η. In (5.1), (5.2) the parasitic state is defined as $\eta = z-u$ resulting into the following representation

$$\dot{y}_p = a_p y_p + 2\eta + u \tag{5.4}$$

$$\mu\dot{\eta} = -\eta - \mu\dot{u} \qquad (5.5)$$

where the "dominant" part (5.4) and "parasitic" part (5.5) of the plant appear explicitly.

If we apply to the plant with parasitics (5.4), (5.5) the same adaptive law which we would have applied to the plant without parasitics, that is if we use the control

$$u = -K(t)y_p + r(t) \qquad (5.6)$$

and the adaptive law

$$\dot{K} = \gamma e y_p \qquad \gamma > 0 \qquad (5.7)$$

we obtain

$$\dot{e} = -a_m e - (K(t)-K^*)(e+y_m) + 2\eta \qquad (5.8)$$

$$\mu\dot{\eta} = -\eta + \mu[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m) + 2K\eta + Kr-\dot{r}] \qquad (5.9)$$

$$\dot{K} = \gamma e(e+y_m) \qquad (5.10)$$

where

$$e \triangleq y_p - y_m, \qquad K^* \triangleq a_m + a_p. \qquad (5.11)$$

The existing theory of adaptive control [33,39] guarantees stability properties for the case without parasitics, $\mu=0$, when (5.8), (5.9), and (5.10) reduce to

$$\dot{\bar{e}} = -a_m\bar{e} - (\bar{K}(t)-K^*)(\bar{e}+y_m) \qquad (5.12)$$

$$\dot{\bar{K}} = \gamma\bar{e}(\bar{e}+y_m). \qquad (5.13)$$

<u>Lemma 5.2.1</u>: For any bounded initial conditions $\bar{e}(0)$, $\bar{K}(0)$ the solution $\bar{e}(t)$, $\bar{K}(t)$ of (5.12),(5.13) is uniformly bounded and $\lim_{t\to\infty} \bar{e}(t) = 0$, $\lim_{t\to\infty} \bar{K}(t) = K_s$ where

constant K_s is in general a function of $\bar{e}(0)$, $\bar{K}(0)$. Furthermore, if $r(t)$ is sufficiently rich then $\lim_{t \to \infty} K(t) = K^*$, independent of $\bar{e}(0)$, $\bar{K}(0)$.

The above example illustrates some of the robustness questions to be answered in this section. Given that the adaptive system without parasitics, in this case (5.12), (5.13), possesses properties such as in Lemma 5.2.1, how will these properties be altered by the parasitics that is, what are the stability properties of (5.8) to (5.10)? Which modification of the adaptive law would help to preserve some of the desirable properties? The perturbation parameter μ provides us with a means to answer such questions in a semi-quantitative way using the orders of magnitude $O(\mu^\nu)$, noting that for μ small, the quantity $O(\mu^\nu)$ is small when $\nu > 0$ and large when $\nu < 0$. The smallness of μ implies that the parasitics are fast and that by neglecting them, $\mu = 0$, we concentrate on the slow, that is the "dominant," part of the plant.

As we shall see a first property to be lost due to parasitics is global stability. In the case of regulation, that is when $y_m = 0$, $r(t) = 0$, the boundedness of the solutions $e(t)$, $K(t)$ and the convergence of $e(t)$ to zero as $t \to \infty$ is preserved, but is not global. It possesses a domain of attraction whose size we describe by estimating the orders of magnitudes of the axes of an ellipsoid $\mathcal{D}(\mu)$. In the tracking problem, when $r(t) \neq 0$ the adaptive system with parasitics such as (5.8) to (5.10) may not converge to or may not even possess an equilibrium. A practical goal is then to guarantee some boundedness properties. We show that a redesign, which may sacrifice some properties of the ideal system without parasitics, results in the convergence from any point in $\mathcal{D}(\mu)$ to a disk $\mathcal{B}(\mu)$ around the origin in the e, η-plane. The design objective is then to make $\mathcal{D}(\mu)$ as large as possible and $\mathcal{B}(\mu)$ as small as possible.

Let us illustrate this discussion by analyzing the regulation problem and the tracking problem for the example (5.1) to (5.3).

a. Regulation: In the regulation problem expressions (5.8) to (5.10) become

$$r(t) = 0,\ y_m(t) = 0,\quad e(t) = y_p(t) \tag{5.14}$$

$$\dot{y}_p = a_p y_p + u + 2\eta \tag{5.15}$$

$$\mu\dot{\eta} = -\eta - \mu\dot{u} \tag{5.16}$$

$$u = -K(t)y_p \tag{5.17}$$

$$\dot{K} = \gamma y_p^2 \tag{5.18}$$

and the objective is to drive y_p to zero despite the presence of parasitics while assuring that all the signals in the closed loop system (5.15) to (5.18) remain bounded. It is important to note that the open loop system (5.15), (5.16) might not be stabilizable by constant gain output feedback for a given value of μ. If this is the case then there is no hope that the adaptive controller (5.17), (5.18) will stabilize the equilibrium of (5.15), (5.16). The following lemma characterizes parasitics for which a linear output stabilizing feedback law exists.

Lemma 5.2.2: There exists a $\mu_1 > 0$ and a constant K_o such that for all $\mu \in (0,\mu_1]$ the system (5.15), (5.16) with the feedback law

$$u = -K_o y_p \tag{5.19}$$

is an asymptotically stable closed-loop system. Furthermore,

$$\mu_1 < \frac{1}{2a_p} \tag{5.20}$$

and

$$\frac{1}{\mu} - a_p > K_o > a_p \tag{5.21}$$

We now establish the stability properties of the adaptive control system (5.15) to (5.18) for $\mu < \mu_1$.

<u>Theorem 5.2.1</u>: There exists $\mu^* < \mu_1$ and positive numbers $\alpha < 1/2$, c_1, c_2 such that for $\mu \in (0,\mu^*]$ any solution $y_p(t)$, $\eta(t)$, $K(t)$ of (5.15) to (5.18) starting from the set

$$\mathcal{D}(\mu) = \{y_p, \eta, K: |y_p| + |K| < c_1\mu^{-\alpha},\ |\eta| < c_2\mu^{-\alpha-1/2}\} \tag{5.22}$$

is bounded and $y_p \to 0$, $\eta \to 0$, $K(t) \to$ constant as $t \to \infty$.

<u>Proof</u>: Let $K_1 > a_p$ be a constant and consider the function

$$V(y_p,\eta,K) = \frac{y_p^2}{2} + \frac{(K-K_1)^2}{2\gamma} + \frac{\mu}{2}(\eta + 2y_p)^2 \tag{5.23}$$

Observe that for each $\mu > 0$, $c > 0$, $\alpha > 0$ the equality

$$V(y_p,\eta,K) = c\mu^{-2\alpha} \tag{5.24}$$

defines a closed surface $\mathcal{S}(\mu, \alpha, c)$ in R^3. The derivative of V along the solution of (5.5) to (5.18) is

$$\dot{V} = -(K_1 - a_p)y_p^2 - \eta^2 + \mu(\eta + 2y_p)(\gamma y_p^3 + Ka_p y_p + 2a_p y_p - 2Ky_p - K^2 y_p + 2K\eta) \tag{5.25}$$

A detailed analysis of (5.25) shows that there exist constants $\alpha < 1/2$, c, μ^* such that $\dot{V} \le 0$ for each $\mu \in (0,\mu^*]$ and all y_p, η, K enclosed in $\mathcal{S}(\mu, \alpha, c)$.

Moreover $\dot{V} = 0$ only at the equilibrium $y_p = 0$, $\eta = 0$, $K =$ constant. The same analysis shows that there exist positive constants c_1, c_2 such that the set

$$\mathcal{D}(\mu) = \{y_p, \eta, K: |y_p| + |K| < c_1\mu^{-\alpha},\ |\eta| < c_2\mu^{-\alpha-1/2}\} \tag{5.26}$$

is enclosed by the surface $S(\mu, \alpha, c)$ and any solution of (5.15) to (5.18) starting from $\mathcal{D}(\mu)$ remains inside $S(\mu, \alpha, c)$. Furthermore inside $S(\mu, \alpha, c)$ $\dot{V}$ is a non-increasing function of time which is bounded from below and hence converges to a finite value V_∞. Since $\ddot{V}$ is bounded, $\dot{V}$ is uniformly continuous for all y_p, η, K enclosed in $S(\mu, \alpha, c)$ and therefore $\lim_{t\to\infty} \dot{V} = 0$ i.e. $y_p \to 0, \eta \to 0$ and $K \to$ constant as $t \to \infty$.

<u>Remark 5.2.1</u>: It can also be shown that increasing adaptive gain γ for a fixed μ reduces the size of the domain $\mathcal{D}(\mu)$ and the stability properties of Theorem 5.2.1 can no longer be guaranteed if $\gamma \geq O(1/\mu)$.

<u>Remark 5.2.2</u>: As $\mu \to 0$, domain $\mathcal{D}(\mu)$ becomes the whole space R^3, that is the adaptive regulation problem (5.15) to (5.18) is well posed with respect to parasitics.

<u>Remark 5.2.3</u>: Theorem 5.2.1 is more than a local result because it shows that given any bounded initial condition $y_p(0)$, $\eta(0)$, $K(0)$, there always exists μ^* such that for each $\mu \in (0,\mu^*]$ the solution of (5.15) to (5.18) is bounded and $y_p \to 0$, $\eta \to 0$, $K \to$ constant as $t \to \infty$.

<u>Remark 5.2.4</u>: Since Theorem 5.2.1 is only a sufficient condition it is of interest to examine whether the stability properties of Lemma 5.2.1 are indeed lost for initial conditions outside the set (5.22). From Lemma 5.2.2 and the fact that K(t) is nondecreasing, it can be seen that instability occurs if $K(t_o) > \frac{1}{\mu} - a_p$.

As an illustration of the stability properties established by Theorem 5.2.1, simulation results for (5.15) to (5.18) with $a_p = 4$ and different values of μ, γ and initial conditions are plotted in Figs. 5.1 to 5.4. In addition to $y_p(t)$ also V with $K_1 = 7$ is plotted against time to show whether all the signals in the closed loop remain bounded. In Figs. 5.1a,b where $\mu = 0.05$, $\gamma = 5$, $y_p(0) = 1.0$, $\eta(0) = 1.0$, $K(0) = 3$, the objective of the regulator is achieved since $y_p \to 0$ and V is bounded. Increasing $y_p(0)$ from 1 to 2.4 and keeping all the other conditions the same as in Fig. 5.1, the regulator fails its objective and $y_p \to \infty$ as shown in Fig. 5.2. With the same initial conditions as in Fig. 5.1a,b but with $\mu = 0.07$ instead of $\mu = 0.05$, $y_p \to \infty$ as indicated in Fig. 5.3. Figure 5.4 shows the effect of increasing the adaptive gain γ. With the same initial conditions as in Fig. 5.1a,b, but with $\gamma = 30$ instead of $\gamma = 5$ regulation fails and $y_p \to \infty$.

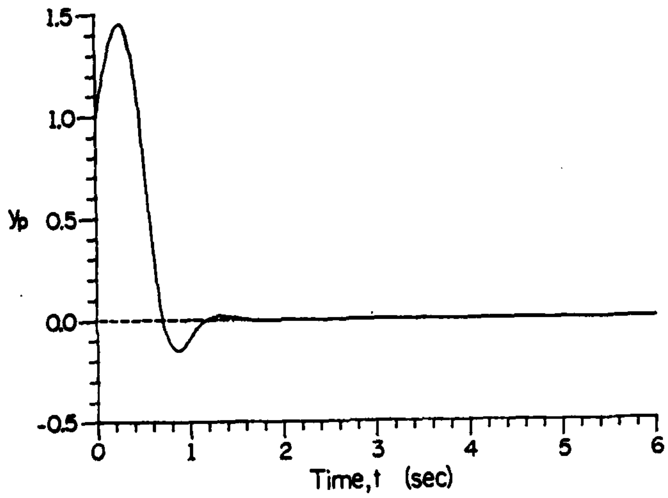
b. <u>Tracking</u>: Returning now to the tracking problem we note that for a general $r(t) \neq 0$ the system (5.8) to (5.10) need not possess an equilibrium. The best we can expect to achieve in this case is to guarantee that the solutions starting in $\mathcal{D}(\mu)$ remain bounded and converge to a disk $\mathcal{B}(\mu)$ around the origin in the e, η-plane.

To prove such a result we modify the adaptive law (5.10) as

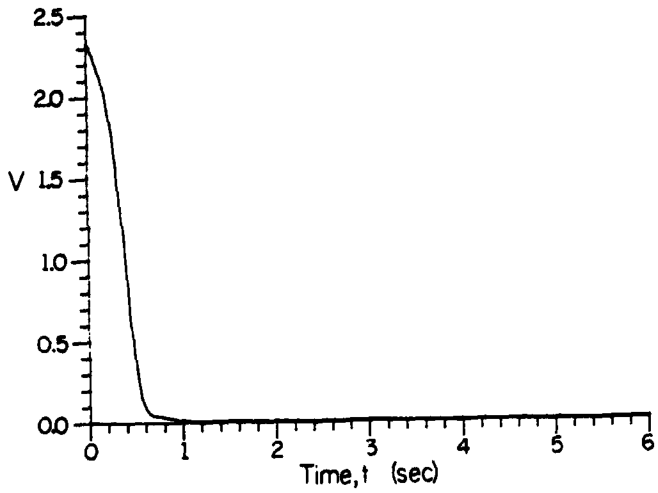
$$\dot{K} = -\sigma K + \gamma e(e + y_m) \tag{5.27}$$

where σ is a positive design parameter. In view of (5.27) the equations describing the stability properties of the tracking problem in the presence of parasitics are

$$\dot{e} = -a_m e - (K(t) - K^*)(e + y_m) + 2\eta \tag{5.28}$$



(a)



(b)

Fig. 5.1 Adaptive regulation results for $\mu = 0.05$, $y_p(o) = 1$, $\eta(o) = 1$, $K(o) = 3$ and $\gamma = 5$.

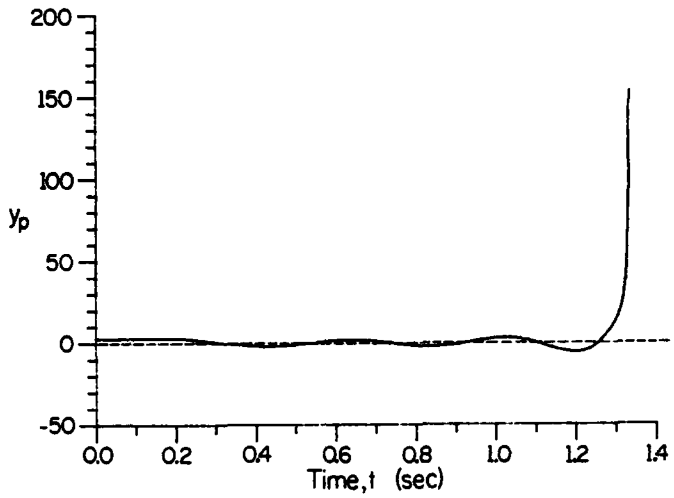


Fig. 5.2 Adaptive regulation results for $\mu = 0.05$, $y_p(o) = 2.4$, $\eta(o) = 1$, $K(o) = 3$ and $\gamma = 5$.

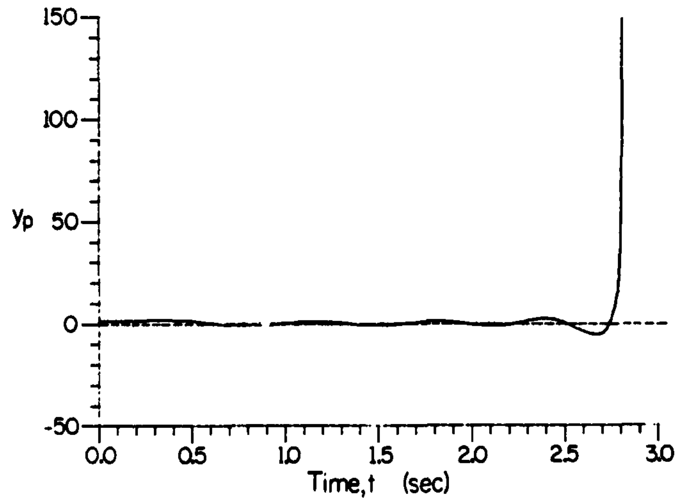


Fig. 5.3 Adaptive regulation results for $\mu = 0.07$, $y_p(o) = 1$, $\eta(o) = 1$, $K(o) = 3$ and $\gamma = 5$.

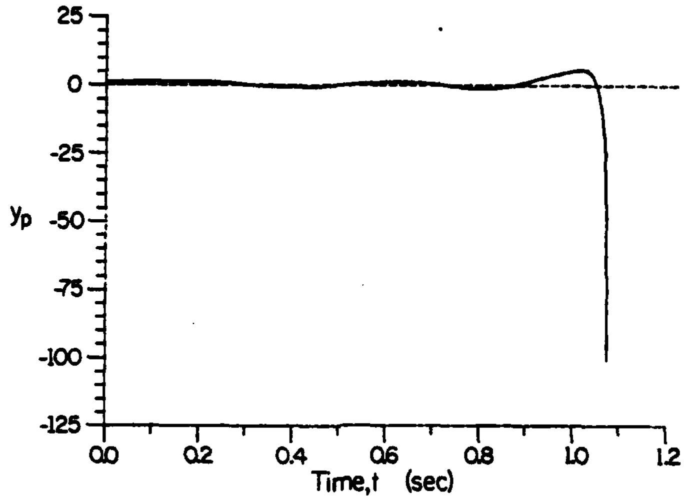


Fig. 5.4 Adaptive regulation results for $\mu = 0.05$, $y_p(o) = 1$, $\eta(o) = 1$, $K(o) = 3$ and $\gamma = 30$.

$$\mu\dot{\eta} = -\eta + \mu[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m) + 2K\eta + Kr - \dot{r}] \tag{5.29}$$

$$\dot{K} = -\sigma K + \gamma e(e+y_m) \tag{5.30}$$

Theorem 5.2.2: Let the reference input r(t) satisfy

$$|r(t)| < r_1 \,, \quad |\dot{r}(t)| < r_2 \quad \forall\, t > 0 \tag{5.31}$$

where r_1, r_2 are given positive constants. Then there exist positive constants t_1, μ^*, σ, $\alpha < 1/2$, c_1 to c_5 such that for $\mu \in (0,\mu^*]$ every solution of (5.28) to (5.30) starting at t = 0 from the set

$$\mathcal{D}(\mu) = \{e,K,\eta: |e| + |K| < c_1\mu^{-\alpha} \,,\ |\eta| < c_2\mu^{-\alpha-1/2}\} \tag{5.32}$$

enters the residual set

$$\mathcal{D}_o(\mu) = \{e,\eta,K: (e,\eta) \in \mathcal{B}(\mu),\ K \in \mathcal{K}\} \tag{5.33}$$

where

$$\mathcal{B}(\mu) = \{e,\eta: |e| + |\eta| \le c_3\sqrt{\sigma}\} \text{ and } \mathcal{K} = \{K: |K| \le c_4\} \tag{5.34}$$

at $t = t_1$ and remains in $\mathcal{D}_o(\mu)$ for all $t > t_1$. Furthermore $c_5 > \sigma > \mu$.

Proof: Choosing the function

$$V(y_p,\eta,K) = \frac{e^2}{2} + \frac{(K-K^*)^2}{2\gamma} + \frac{\mu}{2}(\eta+2e)^2 \tag{5.35}$$

we can see that for each $\mu > 0$, $c_o > 0$, $\alpha > 0$ the equality

$$V(e,\eta,K) = c_o\mu^{-2\alpha} \tag{5.36}$$

defines a closed surface $S(\mu,\alpha,c_o)$ in R^3 space. The derivative of V along the solution of (5.28) to (5.30) is

$$\dot{V} = -a_m e^2 - \frac{\sigma}{\gamma}K(K-K^*) - \frac{\eta^2}{2} + \mu(\eta + 2e)[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m)$$

$$+ 2K\eta + Kr - 2a_m e - \dot{r} - 2(K-K^*)(e+y_m) + 2\eta] \qquad (5.37)$$

Hence

$$\dot{V} \le -a_m e^2 - \frac{\sigma}{\gamma}\left|K - \frac{K^*}{2}\right|^2 - \frac{\eta^2}{2} + \frac{\sigma}{\gamma}\left|\frac{K^*}{4}\right|^2 + \mu|\eta + 2e|\{|\gamma e(e+y_m)^2|$$

$$+ |K(K-a_p)(e+y_m)| + 2|K\eta| + 2|K\eta| + 2|(K-K^*)(e+y_m)|$$

$$+ 2|\eta| + 2a_m|e| + |K|r_1 + r_2\} \qquad (5.38)$$

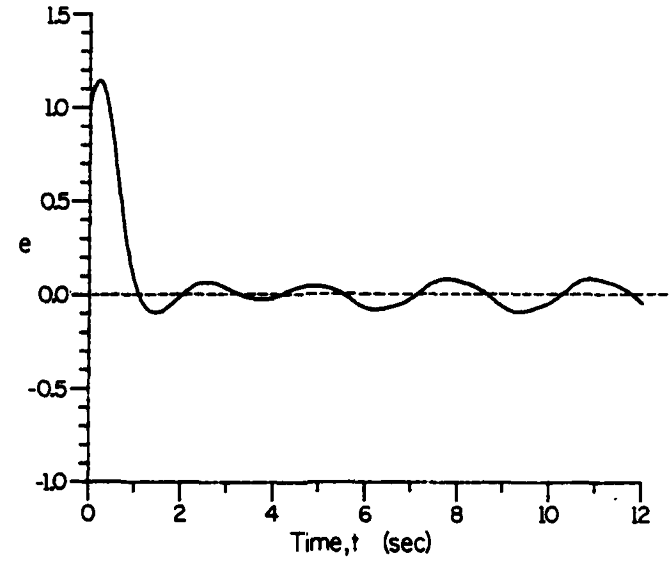
A detailed analysis of (5.38) shows that there exist positive constants c_o, $\alpha < 1/2$, c_1 to c_5, and μ^* such that for all $\mu \in (0,\mu^*]$, and $c_5 > \sigma > \mu$ the sets $\mathcal{D}(\mu)$, $\mathcal{D}_o(\mu)$ given by (5.32) and (5.33), respectively, are enclosed by $S(\mu,\alpha,c_o)$ and $\dot{V} < 0$ everywhere inside $S(\mu,\alpha,c_o)$, except possibly in $\mathcal{D}_o(\mu)$. Furthermore set $\mathcal{D}_o(\mu)$ is closed and bounded, $\mathcal{D}_o(\mu) \subset \mathcal{D}(\mu)$ and $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ is a non-empty set. Thus every solution of (5.28), (5.29), (5.30) starting at $t=0$ from $\mathcal{D}_o(\mu)$ will remain in $\mathcal{D}_o(\mu)$. Also every solution starting at $t=0$ from $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ will enter $\mathcal{D}_o(\mu)$ at $t=t_1$ and remain in $\mathcal{D}_o(\mu)$ thereafter.

<u>Remark 5.2.5</u>: Constants c_i $i=0,1,\ldots,4$ depend on r_1 and r_2 which characterize the magnitude and frequency content of the reference input signal. A further analysis of (5.38) indicates that for a given μ an increase in r_1 or r_2 can no longer guarantee that $\dot{V} < 0$ everywhere in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$. For this reason our formulation excludes high frequency or high amplitude reference input signals such as square or random waveforms, the traditional favorites of the adaptive control literature.

<u>Remark 5.2.6</u>: It can also be shown that increasing the adaptive gain γ for given μ, r_1 and r_2 reduces the size of the domain $\mathcal{D}(\mu)$. For $\gamma \geq O(1/\mu)$ the stability properties of Theorem 5.2.2 can no longer be guaranteed.

<u>Remark 5.2.7</u>: The use of σ is found to be essential in obtaining sufficient conditions for boundedness in the presence of parasitics. However, in the absence of parasitics ($\mu = 0$), $\sigma > 0$ causes an output error of $O(\sqrt{\sigma})$. This is a trade-off between boundedness of all signals in the presence of parasitics and the loss of exact convergence of the output error to zero in the absence of parasitics. The size of σ reflects our ignorance about μ. If an upper bound of μ is known σ can be set equal to this upper bound. For high frequency parasitics μ is small and therefore σ can be small.

It is of interest to examine whether for initial conditions outside the set $\mathcal{D}(\mu)$ we can lose boundedness. Simulation results with $a_p = 4$, $a_m = 3$, and $\gamma = 5$ are summarized in Figs. 5.5 to 5.12. Plots of the output error and the function V versus time are obtained for different initial conditions, μ, σ and reference input characteristics. In Fig. 5.5a,b the output error e and function V are plotted against time for $\mu = 0.01$, $e(0) = 1$, $\eta(0) = 1$, $K(0) = 3$, $\sigma = 0.06$, and $r(t) = 3 \sin 2t$. The output error decreases and remains close to zero and function V is strictly decreasing for $V > 0.05$, but $\dot{V}$ changes sign in the region $V < 0.05$ as shown in Fig. 5.5b. Keeping the same conditions as in Fig. 5.5a,b, but increasing μ from 0.01 to 0.05 we can still achieve similar results as shown in Fig. 5.6a,b. However, in this case the steady state error is larger and $\dot{V}$ changes sign for $V < 0.4$. Increasing the value of μ from 0.05 to 0.08 the output error becomes unbounded for all $\sigma \geq 0$ as indicated in Fig. 5.7. The effects of the input characteristics are summarized in Figs. 5.8 to 5.10. In Fig. 5.8, $\mu = 0.05$, $e(0) = 1$, $\eta(0) = 1$, $K(0) = 3$, $\sigma = 0$, and



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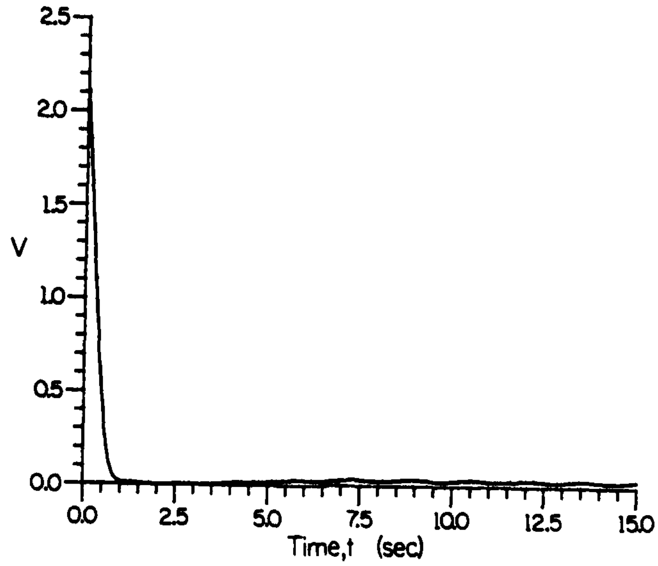
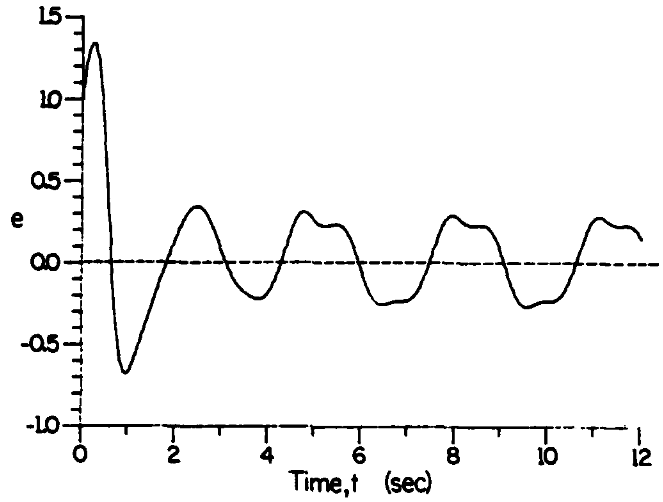
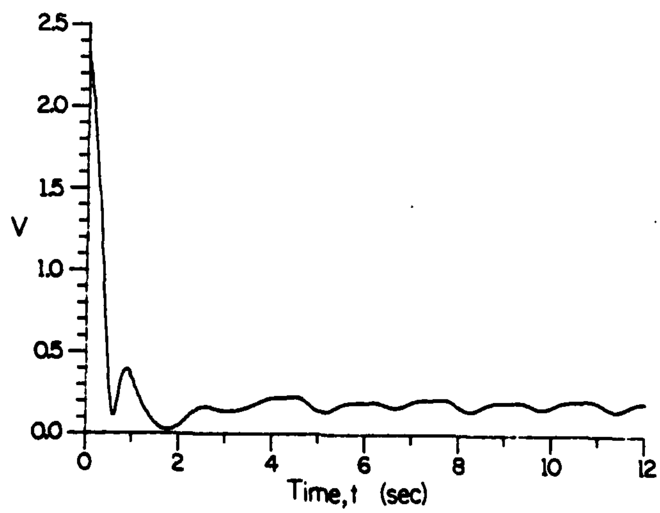


(b)

Fig. 5.5 Tracking results for $\mu = 0.01$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0.06$ and $\gamma(t) = 3 \sin 2t$.



(a)



(b)

Fig. 5.6 Tracking results for $\mu = 0.05$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0.06$ and $r(t) = 3\sin 2t$.

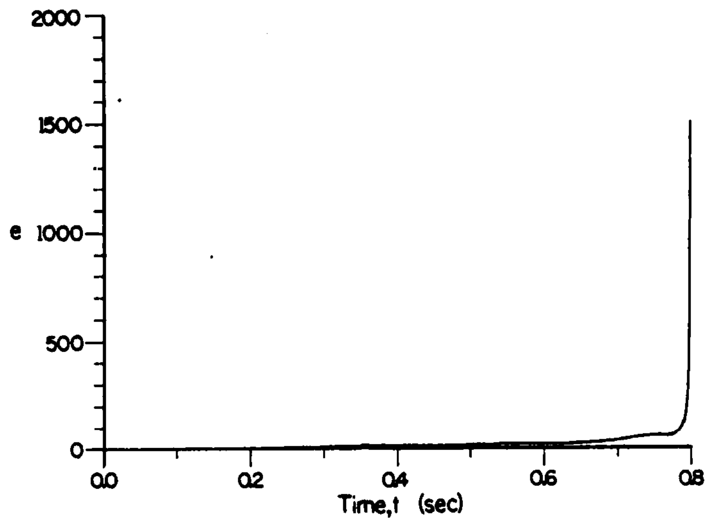


Fig. 5.7 Tracking results for $\mu = 0.08$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma \geq 0.0$ and $r(t) = 3\sin 2t$.

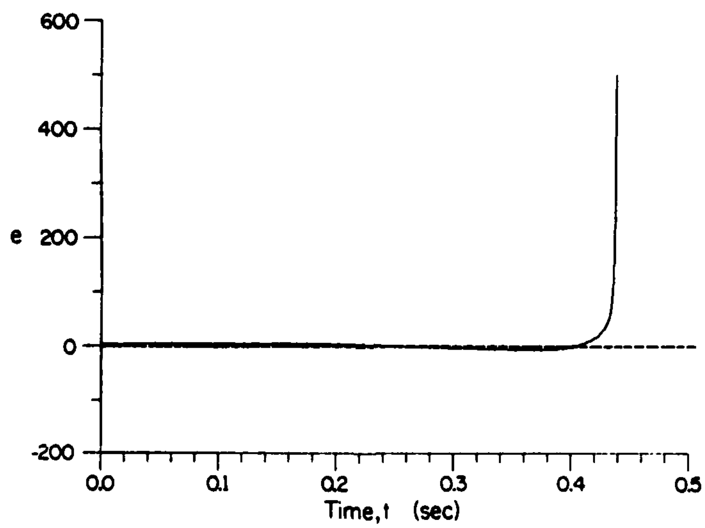


Fig. 5.8 Tracking results for $\mu = 0.05$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0$ or 0.06 and $r(t) = 3\sin 10t$.

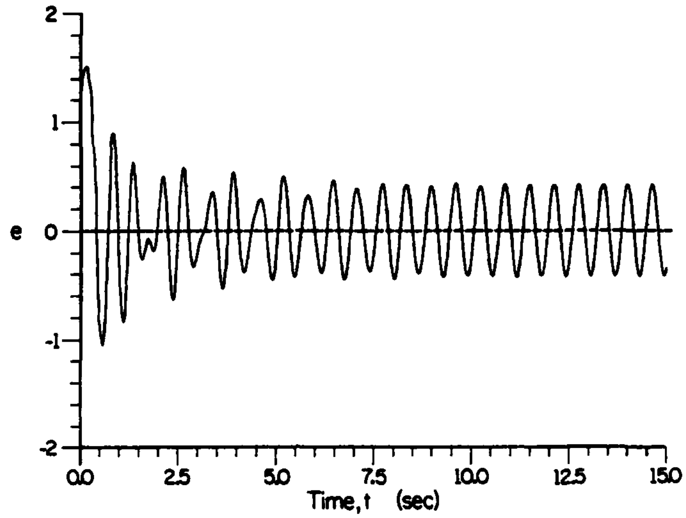


Fig. 5.9 Tracking results for $\mu = 0.05$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0.08$ and $r(t) = 3\sin 10t$.

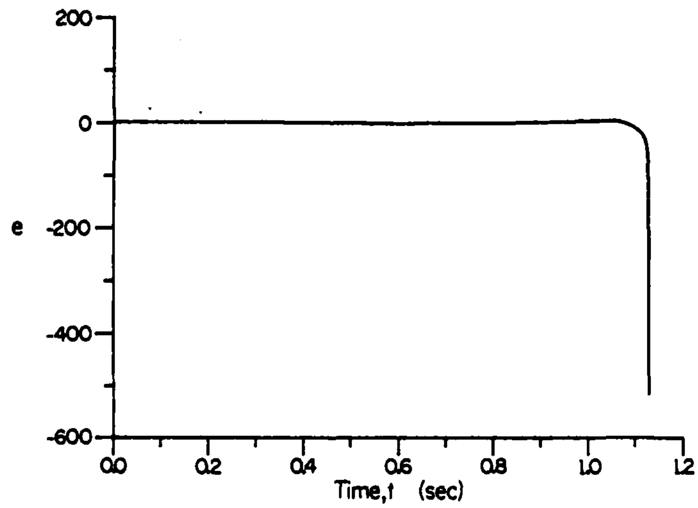


Fig. 5.10 Tracking results for $\mu = 0.05$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0$ or 0.06 and $r(t) = 15\sin 2t$.

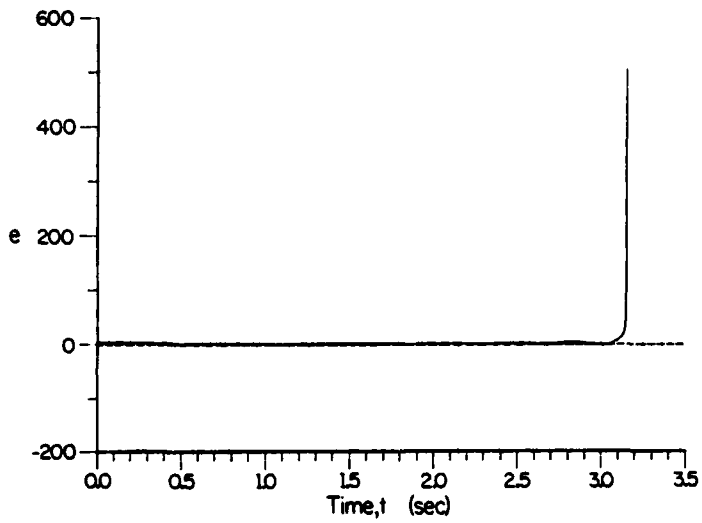
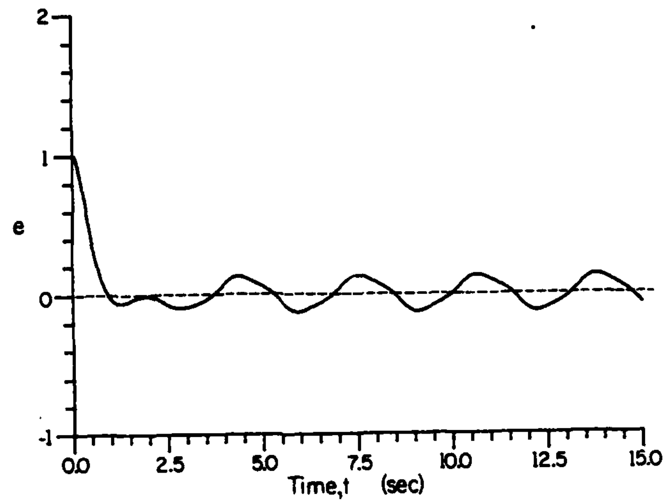
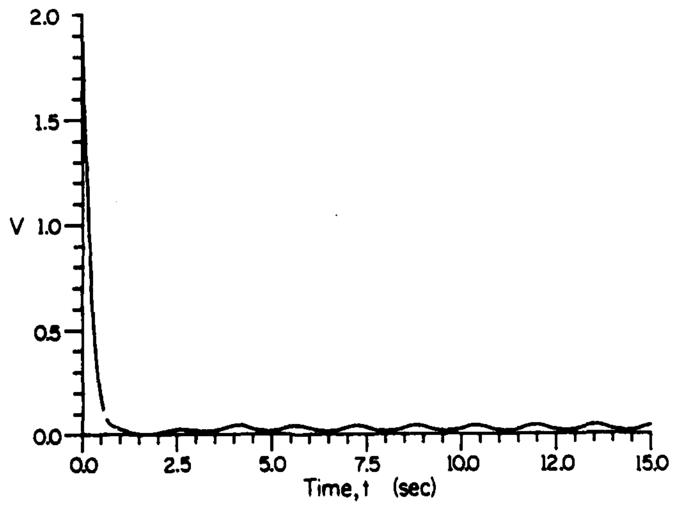


Fig. 5.11 Tracking results for $\mu = 0.05$, $e(o) = 2.5$, $\eta(o) = 1$, $K(o) = 3$, $\sigma \geq 0$ and $r(t) = 3\sin 2t$.



(a)



(b)

Fig. 5.12 Tracking results for $\mu = 0.$, $e(o) = 1$, $\eta(o) = 1$, $K(o) = 3$, $\sigma = 0.08$ and $r(t) = 3\sin 2t$.

$r(t) = 3\sin 10t$ results into an unbounded output error due to the increase of the frequency of $r(t)$ from 2 to 10. The same instability result has been observed for $\sigma = 0.02$, 0.06. However, for $\sigma = 0.08$ the output error became bounded as shown in Fig. 5.9 indicating the beneficial effects of σ when parasitics are present. The effect of the amplitude of the reference input $r(t)$ is shown in Fig. 5.10. With $\mu = 0.05$, $\sigma = 0$, 0.06 and the same initial conditions as before but with $r(t) = 15\sin 2t$ the output error goes unbounded. Figure 5.11 shows the effect of initial conditions on boundedness. By increasing $e(0)$ from 1 to 2.5 and keeping $\mu = 0.05$, $\eta(0) = 1$, $K(0) = 3$, and $r(t) = 3\sin 2t$ the output error becomes unbounded for all $\sigma \geq 0$. In Fig. 5.12a,b we show the loss of exact convergence of the output error to zero in the absence of parasitics ($\mu=0$) due to the design parameter σ.

5.3. Reduced Order Adaptive Control Problem

We now consider the general problem of adaptive control of a SISO time-invariant plant of order $n+m$ where n is the order of the dominant part of the plant and m is the order of the parasitics. The plant is assumed to possess slow and fast parts and is represented in the explicit singular perturbation form

$$\dot{x} = A_{11}x + A_{12}z + b_1u \tag{5.40}$$

$$\mu\dot{z} = A_{21}x + A_{22}z + b_2u\ , \qquad \mathrm{Re}\lambda(A_{22}) < 0 \tag{5.41}$$

$$y = c_o x \tag{5.42}$$

where x, z are n and m vectors respectively and u, y are the scalar input and output of the plant respectively. In this representation it is assumed that the

parasitics are only weakly observable from (5.42), that is, the dependence of y on the parasitic modes is $O(\mu)$. When parasitics are strongly observable, $y = c_1x + c_2z$, constant output feedback can lead to instability in general [40] and this case is of no interest to us at this moment. However, the same filtering technique employed in Section 4.4 can be used to filter strongly observable parasitics and obtain an output of the form (5.42). In (5.41) state z is formed of a "fast transient" and a "quasi-steady state" defined as the solution of (5.41) with $\mu\dot{z} = 0$. This motivates the definition of the fast parasitic state as

$$\eta = z + A_{22}^{-1}(A_{21}x + b_2u). \tag{5.43}$$

Defining

$$\begin{aligned} &A_o = A_{11} - A_{12}A_{22}^{-1}A_{21}, \quad b_o = b_1 - A_{12}A_{22}^{-1}b_2, \quad A_1 = A_{22}^{-1}A_{21}A_o \\ &A_2 = A_{22}^{-1}A_{21}b_o, \quad A_3 = A_{22}^{-1}A_{21}A_{12}, \quad A_4 = A_{22}^{-1}b_2 \end{aligned} \tag{5.44}$$

and substituting (5.43) into (5.40), (5.41) we obtain a representation of (5.40), (5.41), (5.42) with the dominant part (5.45) and the parasitic part (5.46) appearing explicitly

$$\dot{x} = A_ox + b_ou + A_{12}\eta \tag{5.45}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu(A_1x + A_2u + A_3\eta + A_4\dot{u}) \tag{5.46}$$

$$y = c_ox\,. \tag{5.47}$$

The output y of the system (5.45) to (5.47) is required to track the output y_m of an n-th order reference model

$$\dot{x}_m = A_m x_m + b_m r \tag{5.48}$$

$$y_m = c_m^T x_m \tag{5.49}$$

whose transfer function $W_m(s)$

$$W_m(s) = c_m^T(sI - A_m)^{-1} b_m = K_m \frac{Z_m(s)}{R_m(s)} \tag{5.50}$$

is chosen to be strictly positive real and r(t) is a uniformly bounded reference input signal.

The reduced-order plant obtained by setting $\mu = 0$ in (5.45)-(5.47) is assumed to satisfy the following conditions:

(i) The triple (A_o, b_o, c_o) is completely controllable and observable.

(ii) In the transfer function

$$W_o(s) = c_o^T(sI - A_o)^{-1} b_o \triangleq K_p \frac{N(s)}{D(s)} \tag{5.51}$$

N(s) is a monic Hurwitz polynomial of degree n-1 and D(s) is a monic polynomial of degree n. For ease of exposition we assume that $K_p = K_m = 1$.

The controller structure has the same form as that used in [7] for the parasitic -free plant that is for $\mu = 0$ in (5.45) to (5.47). In this controller the plant input u and measured output y are used to generate a (2n-2) dimensional auxiliary vector v as

$$\begin{aligned} \dot{v}_1 &= \Lambda v_1 + gu \\ W_1 &= c^T v_1 \end{aligned} \tag{5.52}$$

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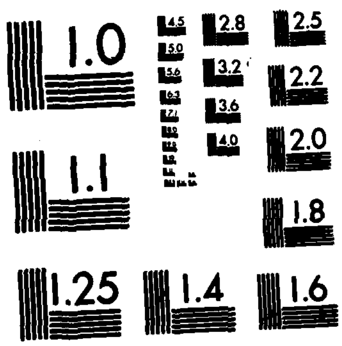

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$$\dot{x}_m = A_m x_m + b_m r \tag{5.48}$$

$$y_m = c_m^T x_m \tag{5.49}$$

whose transfer function $W_m(s)$

$$W_m(s) = c_m^T(sI - A_m)^{-1} b_m = K_m \frac{Z_m(s)}{R_m(s)} \tag{5.50}$$

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The reduced-order plant obtained by setting $\mu = 0$ in (5.45)-(5.47) is assumed to satisfy the following conditions:

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$$W_o(s) = c_o^T(sI - A_o)^{-1} b_o \triangleq K_p \frac{N(s)}{D(s)} \tag{5.51}$$

$N(s)$ is a monic Hurwitz polynomial of degree $n-1$ and $D(s)$ is a monic polynomial of degree n. For ease of exposition we assume that $K_p = K_m = 1$.

The controller structure has the same form as that used in [7] for the parasitic -free plant that is for $\mu = 0$ in (5.45) to (5.47). In this controller the plant input u and measured output y are used to generate a $(2n-2)$ dimensional auxiliary vector v as

$$\begin{aligned} \dot{v}_1 &= \Lambda v_1 + gu \\ W_1 &= c^T v_1 \end{aligned} \tag{5.52}$$

$$\dot{v}_2 = \Lambda v_2 + gy$$
$$W_2 = d_o y + d^T v_2 \tag{5.53}$$

where Λ is an (n-1)x(n-1) stable matrix and (Λ, g) is a controllable pair. The plant input is given by

$$u = r + \theta^T \omega \tag{5.54}$$

where $\omega^T = [v_1^T, y, v_2^T]$ and $\theta(t) = [c^T(t), d_o(t), d^T(t)]^T$ is a (2n-1) dimensional adjustable parameter vector. It has been shown in [7] that a constant vector θ^* exists such that for $\theta(t) = \theta^*$ the transfer function of the parasitic-free plant (5.51) with controller (5.52) to (5.54) matches that of the model (5.50).

If we apply to the plant with parasitics (5.45) to (5.47) the controller described by (5.52) to (5.54) we obtain the following set of equations for the overall feedback system

$$\begin{bmatrix} \dot{x} \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} A_o & 0 & 0 \\ 0 & \Lambda & 0 \\ gc_o & 0 & \Lambda \end{bmatrix} \begin{bmatrix} x \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} b_o \\ g \\ 0 \end{bmatrix} (\theta^T \omega + r) + \begin{bmatrix} A_{12} \\ 0 \\ 0 \end{bmatrix} \eta \tag{5.55}$$

$$\mu \dot{\eta} = A_{22}\eta + \mu(A_1 x + A_2 \theta^T \omega + A_2 r + A_3 \eta + A_4 \dot{\theta}^T \omega + A_4 \theta^T \dot{\omega} + A_4 \dot{v}) \tag{5.56}$$

$$y = c_o x \tag{5.57}$$

Introducing θ^*, $Y^T = [x^T, v_1^T, v_2^T]$ and

$$A_c = \left[\begin{array}{c:c:c} A_o + d_o^* b_o c_o & b_o c^{*T} & b_o d^{*T} \\ \hdashline g d_o^* c_o & \Lambda + g c^{*T} & g d^{*T} \\ \hdashline g c_o & 0 & \Lambda \end{array}\right], \quad b_c = \left[\begin{array}{c} b_o \\ \hdashline g \\ \hdashline 0 \end{array}\right] \tag{5.58}$$

we rewrite (5.55) to (5.56) in a form convenient for our stability analysis

$$\dot{Y} = A_c Y + b_c((\theta - \theta^*)^T \omega + r) + \bar{A}_{12}\eta \tag{5.59}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu(\bar{A}_1 Y + A_2\theta^T\omega + A_2 r + A_3\eta + A_4\dot{\theta}^T\omega + A_4\theta^T\dot{\omega} + A_4\dot{r}) \tag{5.60}$$

where $\bar{A}_{12} = [A_{12}^T \quad 0 \quad 0]^T$ and $\bar{A}_1 = [A_1^T \quad 0 \quad 0]^T$. An advantage of this form is that for $\theta(t) = \theta^*$ in the parasitic -free case (5.59) becomes a non-minimal representation of the reference model

$$\dot{x}_{mc} = A_c x_{mc} + b_c r, \qquad x_{mc} = [x_m^T, v_{1m}^T, v_{2m}^T]^T. \tag{5.61}$$

The equations for the error $e \triangleq Y - x_{mc}$ can be expressed as

$$\dot{e} = A_c e + b_c(\theta - \theta^*)^T(\bar{e} + \bar{x}_{mc}) + \bar{A}_{12}\eta \tag{5.62}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu[\bar{A}_1(e + x_{mc}) + A_2\theta^T(\bar{e} + \bar{x}_{mc}) + A_2 r + A_3\eta + A_4\dot{\theta}^T(\bar{e} + \bar{x}_{mc})$$

$$+ A_4\theta^T f(\theta, e, \eta, r) + A_4\dot{r}] \tag{5.63}$$

$$e_1 = h^T e = [1 \quad 0 \; . \; . \; 0]e \tag{5.64}$$

where

$$\bar{e} \triangleq [v_1^T, y, v_2^T]^T - [v_{1m}^T, y_m, v_{2m}^T]^T, \quad \bar{x}_{mc} \triangleq [v_{1m}^T, y_m, v_{2m}^T]^T \tag{5.65}$$

$$f(\theta,e,\eta,r) = \begin{bmatrix} \Lambda(e^{(1)}+v_{1m})+gr+g\theta^T(\bar{e}+\bar{x}_{mc}) \\ \Lambda(e^{(2)}+v_{2m})+g(e_1+y_m) \\ c_oA_o(e^{(0)}+x_m)+c_ob_o(r+\theta^T(\bar{e}+\bar{x}_{mc}))+A_{12}\eta \end{bmatrix} \tag{5.66}$$

$$e^{(0)} \triangleq x - x_m,\ e^{(1)} \triangleq v_1 - v_{1m},\ e^{(2)} \triangleq v_2 - v_{2m} \tag{5.67}$$

We now need to design an adaptive law for updating the parameter vector $\theta(t)$. For the parasitic-free case [7] the adaptive law

$$\dot{\theta} = -\Gamma e_1\omega = -\Gamma e_1(\bar{e} + \bar{x}_{mc}),\quad \Gamma = \Gamma^T > 0 \tag{5.68}$$

guarantees that the output error goes to zero as $t \to \infty$ and the signals in the close loop remain bounded for any uniformly bounded reference input $r(t)$. As demonstrated in Section 5.2 for the scalar tracking problem, the best we can expect in the presence of parasitics is to guarantee that the solutions starting in a domain $\mathcal{D}(\mu)$ remain bounded and converge to a disk $\mathcal{B}(\mu)$ around the origin in the e, η plane. To achieve this we modify the adaptive law (5.68) as

$$\dot{\theta} = -\sigma\theta - \Gamma e_1(\bar{e} + \bar{x}_{mc}) \tag{5.69}$$

where σ is a design scalar parameter. The resulting adaptive control system with parasitics is described by

$$\dot{e} = A_c e + b_c(\theta - \theta^*)^T(\bar{e}+\bar{x}_{mc}) + \bar{A}_{12}\eta \tag{5.70}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu[\bar{A}_1(e+x_{mc}) + A_2\theta^T(\bar{e}+\bar{x}_{mc}) + A_2 r + A_3\eta - \sigma A_4\theta^T(\bar{e}+\bar{x}_{mc})$$
$$- A_4(\bar{e}+\bar{x}_{mc})^T\Gamma(\bar{e}+\bar{x}_{mc})e_1 + A_4\theta^T f(\theta,e,\eta,r) + A_4\dot{r}] \tag{5.71}$$

$$e_1 = [1\ 0\ .\ .\ 0]e \tag{5.72}$$

$$\dot{\theta} = -\sigma\theta - \Gamma e_1(\bar{e}+\bar{x}_{mc}). \tag{5.73}$$

Theorem 5.3.1: Let the reference input $r(t)$ satisfy

$$|r(t)| < r_1\ ,\quad |\dot{r}(t)| < r_2 \qquad \forall\ t > 0 \tag{5.74}$$

for some given positive constants r_1, r_2. Then there exists positive constants μ^*, σ, $\alpha < 1/2$, c_1 to c_5 and t_1 such that for each $\mu \in (0,\mu^*]$ every solution of (5.70) to (5.73) starting at $t = 0$ from the set

$$\mathcal{D}(\mu) = \{e,\eta,\theta:\ \|e\| + \|\theta\| < c_1\mu^{-\alpha},\ \|\eta\| < c_2\mu^{-\alpha - 1/2}\} \tag{5.75}$$

enters the residual set

$$\mathcal{D}_o(\mu) = \{e,\eta,\theta:\ (e,\eta) \in \mathcal{B}(\mu),\ \theta \in \mathcal{X}\} \tag{5.76}$$

where

$$\mathcal{B}(\mu) = \{e,\eta:\ \|e\| + \|\eta\| \le c_3\sqrt{\sigma}\},\ \mathcal{X} = \{\theta:\ \|\theta\| \le c_4\} \tag{5.77}$$

at $t = t_1$ and remains in $\mathcal{D}_o(\mu)$ for all $t > t_1$. Furthermore $c_5 > \sigma > \mu$.

Corollary 5.3.1: Assume $r(t) = 0$. Then there exists a μ^* such that for all $\mu \in (0,\mu^*]$ and $\sigma = 0$ in (5.73) any solution $e(t)$, $\eta(t)$, $\theta(t)$ of (5.70) to (5.73) which starts from $\mathcal{D}(\mu)$ given by (5.75) is bounded and $\|e\| \to 0$, $\|\eta\| \to 0$, $\|\theta\| \to$ constant as $t \to \infty$.

Proof of Theorem 5.3.1: Choose the function

$$V(e,\eta,\theta) = \frac{e^T Pe}{2} + \frac{(\theta - \theta^*)^T \Gamma^{-1}(\theta - \theta^*)}{2} + \frac{\mu}{2}[\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T]^T P_1[\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T] \tag{5.78}$$

where P satisfies

$$A_c^T P + PA_c = -qq^T - \epsilon L \tag{5.79}$$

$$Pb_c = h \tag{5.80}$$

for some vector q, matrix $L = L^T > 0$ and $\epsilon > 0$, and P_1 satisfies

$$P_1 A_{22} + A_{22}^T P_1 = -Q_1 , \quad Q_1 = Q_1^T > 0 \tag{5.81}$$

Equations (5.79), (5.80) follow from the fact that $h^T(sI - A_c)b_c$ is strictly positive real [7] and (5.81) follows from the assumption that $\mathrm{Re}\lambda(A_{22}) < 0$.

Observe that for each $\mu > 0$, $d_o > 0$, $\alpha < 1/2$ the equality

$$V(e,\eta,\theta) = d_o \mu^{-2\alpha} \tag{5.82}$$

defines a closed surface $S(\mu,\alpha,d_o)$ in R^{6n+m-2}. The derivative of V along the solution of (5.70) to (5.73) is

$$\dot{V} = -\frac{1}{2} e^T(qq^T + \epsilon L)e - \sigma(\theta - \theta^*)^T\Gamma^{-1}\theta - \frac{1}{2} \eta^T Q_1 \eta + \mu[\eta$$

$$- P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T]^T P_1[\bar{A}_1(e + x_{mc}) + A_2\theta^T(\bar{e} + \bar{x}_{mc}) + A_2 r + A_3\eta$$

$$- \sigma A_4\theta^T(\bar{e} + \bar{x}_{mc}) - A_4(\bar{e} + \bar{x}_{mc})^T\Gamma(\bar{e} + \bar{x}_{mc})e_1 + A_4\theta^T f(\theta, e, \eta, r) + A_4\dot{r}$$

$$- P_1^{-1}A_{22}^{-1T}\bar{A}_{12}^T P(A_c e + b_c(\theta - \theta^*)^T(\bar{e} + \bar{x}_{mc}) + \bar{A}_{12}\eta)] \tag{5.83}$$

Let

$$\lambda_1 = \min \lambda(L), \; \lambda_2 = \min \lambda(\Gamma^{-1}), \quad \lambda_3 = \min \lambda(Q_1) \tag{5.84}$$

Then

$$\dot{V} \le -\frac{1}{2} \lambda_1\|e\|^2 - \sigma\lambda_2[\|\theta\| - \frac{\|\Gamma^{-1}\|}{2\lambda_2} \|\theta^*\|]^2 - \frac{1}{2} \lambda_3\|\eta\|^2 + \frac{\sigma\|\Gamma^{-1}\|^2\|\theta^*\|^2}{4\lambda_2}$$

$$+ \mu\|\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T\|[\alpha_1\|e\|^3 + \alpha_2\|e\|^2 + \alpha_3\|\theta\|^2\|e\| + \alpha_4\|\theta\|\|e\|$$

$$+ \alpha_5\|\theta\|^2 + \alpha_6\|\theta\| + \alpha_7\|\theta\|\|\eta\| + \alpha_8\|\eta\| + \|A_4\|r_2 + \alpha_9] \tag{5.85}$$

where α_1 to α_9 are positive constants determined from r_1, and the norms of the system and the reference model matrices. A detailed analysis of (5.85) shows that there exists positive constants σ, $\alpha < 1/2$, c_i $i = 1, \ldots, 5$ and μ^* such that for all $\mu \in (0, \mu^*]$ and $c_5 > \sigma > \mu$, $\mathcal{D}(\mu)$, $\mathcal{D}_o(\mu)$ defined by (5.75) to (5.77) are enclosed by $S(\mu, \alpha, d_o)$ and $\dot{V} < 0$ everywhere inside $S(\mu, \alpha, d_o)$ except possibly in $\mathcal{D}_o(\mu)$. The set $\mathcal{D}_o(\mu)$ is closed and bounded, $\mathcal{D}_o(\mu) \subset \mathcal{D}(\mu)$ and $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ is a non-empty set. Every solution of (5.70) to (5.73) starting at $t = 0$ from $\mathcal{D}_o(\mu)$ will remain in $\mathcal{D}_o(\mu)$. Since in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$, V is strictly decreasing any solution starting at $t = 0$ from $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ will enter $\mathcal{D}_o(\mu)$ at $t = t_1$ and remain in $\mathcal{D}_o(\mu)$ thereafter.

Proof of Corollary 5.3.1: The proof of Corollary 5.3.1 follows directly from the proof of Theorem 5.3.1 by noting that when $r(t)=0$, $x_m=0$, and $\sigma=0$, the disk $\mathcal{B}(\mu)$ reduces to the origin $e=0$, $\eta=0$, i.e. in (5.77) $c_3=0$.

In Theorem 5.3.1 and Corollary 5.3.1 is assumed that $\mu^* < \mu_1$ where μ_1 is defined in the following lemma.

Lemma 5.3.1: There exists a $\mu_1 > 0$ such that constant output feedback $u = \theta_o^T \omega$ stabilizes (5.55) to (5.57) for all $\mu \in (0,\mu_1]$.

The proof of Lemma 5.3.1 is more complicated than that of Lemma 5.2.1 and can be found in [41] where an explicit expression has been obtained for μ_1.

Remark 5.3.1: The dependence of constants c_i $i=1,\ldots,4$ on r_1, r_2 shows that for a given μ a reference input signal with high magnitude or high frequencies can no longer guarantee that $\dot{V}<0$ everywhere in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$. Such a reference signal introduces frequencies in the input control signal which are in the parasitic range. Thus the control signal is no longer dominantly rich and, hence, it excites the parasitics considerably and leads to instability. This explains the instability phenomena observed by other authors in simulations such as [23] where a square wave was used as a reference input signal.

5.4. Discussion

In this section we have analyzed reduced-order adaptive control schemes in which reference models can match the dominant part of the plant, while the model-plant mismatch is caused by the neglected high frequency parasitic modes. In the presence of parasitics the global stability properties of the parasitic-free schemes can be lost. However, we have shown that in the regulation problem a region of attraction exists for exact adaptive regulation. This

region is a function of the adaptive gains and the speed ratio μ, and as $\mu \to o$, it becomes the whole space. Thus the adaptive regulation problem is well posed with respect to parasitics. In the case of tracking we proposed a more robust adaptive law. The new scheme guarantees the existence of a region of attraction from which all signals converge to a residual set which contains the equilibrium for exact tracking. The dependence of the size of this set on design parameters indicates that a trade-off can be made sacrifizing some of the ideal parasitic-free properties, in order to achieve robustness in presence of parasitics. The crucial effects of the frequency range of parasitics, the adaptive gains and the reference input signal characteristics on the stability properties of adaptive control schemes, explain the undesirable phenomena observed in [22-24]. The results of this section are obtained for a continuous-time SISO adaptive control scheme where the transfer function of the dominant part of the plant has a relative degree of one. The same methodology can be extended to more complicated continuous and discrete-time adaptive control problems.

6. DECENTRALIZED ADAPTIVE CONTROL

6.1. Introduction and Problem Statement

In classical control theory the systems under study are assumed to have only one controller who determines the control actions based on the available information of the system. Systems of this type are called centralized systems.

When control theory is applied to solve problems of large systems, e.g. electric power systems, socioeconomic systems, computer communication networks, etc., an important feature called decentralization often arises. Such systems have several local control stations; at each station, the controller observes only local system outputs and controls only local inputs. All the controllers are involved, however, in controlling the overall large system.

In decentralized control the problem of stability of the overall system becomes very important. It received the attention of many authors in the last few years [42-44], but their studies were focused mainly on systems with known parameters. The operating environment of most large-scale systems is rather poorly known and their parameters cannot be calculated with sufficient accuracy to be used in on-line controllers. Consequently, DACRs (Decentralized Adaptive Controllers) are essential for large-scale systems with unknown parameters.

In this section an approach for the decentralized regulation and tracking of a class of large-scale linear dynamical systems with unknown parameters is developed. Sufficient conditions for decentralized adaptive regulation in a form of algebraic criteria are established which can

guarantee adaptive regulation under certain structural perturbations. In the case of tracking, we propose a decentralized adaptive scheme which guarantees that all the signals of the closed-loop system are bounded and the state error converges to a residual set which contains the equilibrium for exact tracking.

The formal problem statement is as follows: Consider a linear time-invariant system S, which is described as an interconnection of N subsystems $S_1, S_2, \ldots, S_N$ and is represented by

$$\dot{x}_i = A_i x_i + b_i u_i + f_i(x) \tag{6.1}$$

$$f_i(x) = \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j \tag{6.2}$$

$$y_i = h_i^T x_i, \qquad i = 1, 2, \ldots, N \tag{6.3}$$

which for the subsystem S_i : $x_i \in R^{n_i}$ is the state vector, $u_i \in R^1$ is the control variable, $y_i \in R^1$ is the output, and $f_i(x) \in R^{n_i}$ is the interaction vector from the other subsystems to S_i. The parameters A_i, A_{ij}, b_i, and h_i are unknown constant matrices and all the triples (A_i, b_i, h_i) are completely controllable and completely observable.

In this representation the composite system S can be described as

$$\dot{x} = Ax + Bu + f \tag{6.4}$$

$$y = Cx \tag{6.5}$$

where $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$ is the composite state vector, $y = [y_1, y_2, \ldots, y_N]^T$ is the composite output vector, $u = [u_1, u_2, \ldots, u_N]^T$ is the composite control

vector, $f = Hx$ represents the interconnection pattern of the overall system S, and

$$H = \begin{bmatrix} 0 & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & 0 & & & A_{2N} \\ \vdots & & & & \vdots \\ A_{N1} & & & & 0 \end{bmatrix} \tag{6.6}$$

is the interconnection matrix. Furthermore, $A = \mathrm{diag}(A_i)$, $B = \mathrm{diag}(b_i)$, $C = \mathrm{diag}(h_i^T)$, $A \in R^{n\times n}$, $B \in R^{n\times n}$, and $C \in R^{N\times n}$ where $n = \sum_{i=1}^{N} n_i$. The problem is to design LACRs (Local Adaptive Controllers) such that the states of the composite system (6.4), (6.5) are regulated to zero or track the states of a given reference model. Each LACR is associated with only one subsystem and it uses information only from that subsystem. That is the LACRs are not allowed to communicate with each other.

6.2. Adaptive Control Using the State of the Plant

To complement the approach used in Section 6.3 for the DAC (Decentralized Adaptive Control) of a large-scale system composed of SISO subsystems we shall consider here the relatively simple case of a system whose state variables can be measured and vector b_i is known for each subsystem. The reference model M_i whose states are to be tracked by the states of subsystem S_i is

$$\dot{x}_{m_i} = A_{m_i} x_{m_i} + b_i r_i \qquad i = 1,2,\ldots,N \tag{6.7}$$

where $x_{m_i} \in R^{n_i}$, r_i is a piecewise continuous uniformly bounded reference

input and A_{m_i} is a stable matrix. It is further assumed that a vector K_i^* exists such that

$$A_i + b_i K_i^{*T} = A_{m_i}. \tag{6.8}$$

In view of (6.7) the reference model M for the composite system S is

$$\dot{x}_m = A_m x_m + Br \tag{6.9}$$

where $x_m = [x_{m_1}^T, x_{m_2}^T, \ldots, x_{m_N}^T]$, $r = [r_1, r_2, \ldots, r_N]^T$, and $A_m = \mathrm{diag}(A_{m_i})$. The basic idea of the DAC is to determine LAC (Local Adaptive Control) inputs u_i such that the error $e \overset{\Delta}{=} x - x_m$ between the composite system S and reference model M as well as all signals in the closed-loop system remain uniformly bounded. Due to the interconnection matrix H it is not possible to insure $\lim_{t\to\infty} \|e\| = 0$ for any bounded reference input vector r. The maximum we can achieve in this case is the convergence of e to some bounded residual set. In the case of regulation ($r=0$, $x_m=0$) the objective is to find LAC inputs u_i to regulate the states of S to zero.

<u>a. Regulation</u>: In the regulation problem $r=0$, $x_m=0$, the LAC inputs u_i are chosen as [2]

$$u_i = \hat{K}_i^T(t) x_i \tag{6.10}$$

and $\hat{K}(t)$ is adjusted according to the adaptive law

$$\dot{\hat{K}}_i = -\Gamma_i (b_i^T P_i x_i) x_i \tag{6.11}$$

where $\Gamma_i = \Gamma_i^T > 0$, $P_i = P_i^T > 0$, and P_i satisfies the Lyapunov equation

$$P_i A_{mi} + A_{mi}^T P_i = -Q_i, \quad Q_i = Q_i^T > 0. \tag{6.12}$$

The closed loop decoupled subsystems are given by

$$\dot{x}_i = (A_i + b_i \hat{K}_i^T) x_i, \qquad i = 1,2,\ldots,N \tag{6.13}$$

and have the property that if $\hat{K}_i$ is updated according to (6.11) $x_i \to 0$, $\hat{K}_i \to$ constant as $t \to \infty$. Using the LACR (6.10), (6.11) the composite system (6.4) becomes

$$\dot{x} = (A + B\hat{K}^T(t))x + Hx \tag{6.14}$$

where $\hat{K} = \mathrm{diag}(K_i)$, and $P = \mathrm{diag}(P_i)$ is the solution of

$$PA_m + A_m^T P = -Q, \qquad Q = \mathrm{diag}(Q_i). \tag{6.15}$$

The presence of interconnection H can change the stability properties of the decoupled subsystems, and it is necessary to obtain sufficient conditions to guarantee the stability of the overall system S. This is given by the following theorem.

Theorem 6.2.1: If the matrix $G = Q - (PH + H^T P)$ is positive definite then the solution $x(t)$, $K_i(t)$ of (6.11), (6.14) is bounded and $\lim_{t\to\infty} \|x(t)\| = 0$, $\lim_{t\to\infty} \|\hat{K}(t)\| =$ constant.

Proof: Consider the positive definite function

$$\dot{V} = x^T P x + \sum_{i=1}^{N} (\hat{K}_i - K_i^*)^T \Gamma_i^{-1} (\hat{K}_i - K_i^*). \tag{6.16}$$

The time derivative of V along the solution of (6.11), (6.14) is

$$\dot{V} = -x^T(PA_m + A_m^T P)x + 2x^T PB(K-K^*)^T x + 2x^T PHx - 2\sum_{i=1}^{N} (\hat{K}_i - K_i^*)^T (b_i^T P_i x_i) x_i \tag{6.17}$$

where $K^* = \mathrm{diag}(K_i^*)$. Using (6.15) and rearranging (6.17) we have

$$\dot{V} = -x^T(Q - (PH + H^T P))x = -x^T G x. \tag{6.18}$$

If G is positive definite then $\dot{V}$ is negative semidefinite and therefore the solution x, $\hat{K}$ of (6.14), (6.11) is uniformly bounded. This implies that $\ddot{V}$ is uniformly bounded and therefore $\dot{V}$ is uniformly continuous. Since V is a non-increasing function and is bounded from below, it converges to a finite value V_∞. For any bounded initial conditions x(0), $\hat{K}(0)$

$$\lim_{t\to\infty} \int_0^t \dot{V}\,dt = V_\infty - V_o < \infty \tag{6.19}$$

and hence $\lim_{t\to\infty}\|\dot{V}\|=0$, i.e. $\lim_{t\to\infty}\|x\| = 0$ and $\lim_{t\to\infty}\|\hat{K}\| = \text{constant}$.

<u>Lemma 6.2.1</u>: A sufficient condition for G to be positive definite is

$$\min_i \lambda_{min}(Q_i) > 2\|H\| \max_i \max \lambda(P_i). \tag{6.20}$$

<u>Proof</u>: For G to be positive definite it is sufficient to show that

$$x^T(Q - PH - H^TP)x > 0 \tag{6.21}$$

or

$$x^TQx > x^T(PH + H^TP)x \tag{6.22}$$

for all x. Inequality (6.22) is satisfied if

$$\lambda_{min}(Q) > 2\|H\| \lambda_{max}(P). \tag{6.23}$$

Since

$$\lambda_{min}(Q) = \min_i \lambda_{min}(Q_i), \quad \lambda_{max}(P) = \max_i \lambda_{max}(P_i) \tag{6.24}$$

(6.21) follows from (6.23) and (6.24).

<u>Lemma 6.2.2</u>: If there exists a vector K_i^* such that

$$A_i + b_iK_i^{*T} = \frac{A_{m_i}}{\varepsilon}, \qquad i = 1,2,\ldots,N \tag{6.25}$$

where ε is an arbitrarily small positive scalar then the stability properties of Theorem 6.2.1 are guaranteed for any bounded $\|H\|$.

<u>Proof</u>: In view of (6.25) the properties of Theorem 6.2.1 are guaranteed if

$$G = \frac{Q}{\varepsilon} - (PH + H^T P) \tag{6.26}$$

is positive definite. Following the procedure of Lemma 6.2.1 G given by (6.26) is positive definite if

$$\min_i \lambda_{min}(Q_i) > \varepsilon \|H\| \max_i \max \lambda(P_i). \tag{6.27}$$

Since ε can be arbitrarily small (6.27) will always be satisfied for any bounded $\|H\|$.

<u>b. Tracking</u>: For the tracking problem we have chosen the LAC inputs u_i as

$$u_i = \hat{K}_i^T(t)x_i + r_i, \qquad i = 1,2,\ldots,N \tag{6.28}$$

and the adaptive law for adjusting $\hat{K}_i(t)$ as

$$\dot{\hat{K}}_i = -\Gamma_i(b_i^T P_i e_i)x_i - \sigma_i \hat{K}_i(t) \tag{6.29}$$

where $e_i \triangleq x_i - x_{m_i}$ and σ_i is a design positive scalar parameter. Then the closed-loop composite system S becomes

$$\dot{x} = A_m x + B(\hat{K}-K^*)^T x + Br + Hx. \tag{6.30}$$

Subtracting (6.9) from (6.30) we obtain the state error equation

$$\dot{e} = A_m e + B(\hat{K}-K^*)^T x + H(e + x_m) \tag{6.31}$$

which has a persistent input Hx_m due to the interconnections. This input acts as a disturbance in the error equation and therefore the solution of (6.31) may not converge to or may not even possess an equilibrium. The following theorem establishes sufficient conditions for boundedness and convergence of the solution of (6.29), (6.31) to a residual set.

<u>Theorem 6.2.2</u>: If the matrix $G = Q-(PH+H^TP)$ is positive definite then the solution $e(t)$, $\hat{K}_i(t)$ $i=1,2,\ldots,N$ of (6.29), (6.31) is bounded and converges to the residual set

$$\mathcal{D} = \{e,\hat{K}_i : \frac{\|e\|^2}{2}\lambda_g + 2\sum_{i=1}^{N}\sigma_i\lambda_i(\|\hat{K}_i\| - \frac{\|K_i^{*T}\Gamma_i^{-1}\|}{2\lambda_i})^2 < d_o\} \tag{6.32}$$

where

$$d_o = \frac{2\lambda_p^2\|H\|^2}{\lambda_g}\sup_{t}\|x_m\|^2 + \sum_{i=1}^{N}\frac{\sigma_i}{2\lambda_i}\|K_i^{*T}\Gamma_i^{-1}\|^2 \tag{6.33}$$

and

$$\lambda_g = \min\lambda(G), \quad \lambda_i = \min\ \lambda(\Gamma_i^{-1}), \quad \lambda_p = \max_i\max\ \lambda(P_i). \tag{6.34}$$

<u>Proof</u>: Consider the positive definite function

$$V = e^TPe + \sum_{i=1}^{N}(\hat{K}_i-K_i^*)^T\Gamma_i^{-1}(\hat{K}_i-K^*) \tag{6.35}$$

where P satisfies (6.15) and Γ_i is a positive definite matrix. The time derivative of V along the solution of (6.29), (6.31) is

$$\dot{V} = -e^T[Q-(PH+H^TP)]e + 2e^TPHx_m + 2\sum_{i=1}^{N}\sigma_iK_i^{*T}\Gamma_i^{-1}\hat{K}_i - \sigma_i\hat{K}_i^T\Gamma_i^{-1}\hat{K}_i. \tag{6.36}$$

Assuming that $G = Q-(PH+H^TP)$ is positive definite and using (6.34) we have

$$\dot{V} \le -\|e\|^2\lambda_g + 2\|e\|\|P\|\|H\|\|x_m\| + \sum_{i=1}^{N}\frac{\sigma_i}{2\lambda_i}\|K_i^{*T}\Gamma_i^{-1}\|^2 - 2\sigma_i\lambda_i(\|K_i\| - \frac{\|K_i^{*T}\Gamma_i^{-1}\|}{2\lambda_i})^2. \tag{6.37}$$

Hence

$$\dot{V} < -\frac{\|e\|^2}{2}\lambda_g - 2\sum_{i=1}^{N}\sigma_i\lambda_i(\|\hat{K}_i\| - \frac{\|\hat{K}_i^{*T}\Gamma_i^{-1}\|}{2\lambda_i})^2 + \frac{2\|P\|^2\|H\|^2\|x_m\|^2}{\lambda_g} + \sum_{i=1}^{N}\frac{\sigma_i}{2\lambda_i}\|K_i^{*T}\Gamma_i^{-1}\|^2. \tag{6.38}$$

From (6.15) and (6.33)

$$\dot{V} < -\frac{\|e\|^2}{2}\lambda_g - 2\sum_{i=1}^{N}\sigma_i\lambda_i\left(\|\hat{K}_i\| - \frac{\|K_i^{*T}\Gamma_i^{-1}\|}{2\lambda_i}\right)^2 + d_o \tag{6.39}$$

which implies that $\dot{V} < 0$ outside the set $\mathcal{D}$. Hence any solution e, $\hat{K}_i$, $i=1,2,\ldots,N$ starting outside the set $\mathcal{D}$ will enter $\mathcal{D}$ in finite time and remain in $\mathcal{D}$ thereafter. Any solution starting at t=0 from $\mathcal{D}$ will remain in $\mathcal{D}$ for all $t \geq 0$.

Remark 6.2.1: The results of Lemma 6.2.1 for the positive definiteness of matrix G applies here too.

Remark 6.2.2: If condition (6.25) of Lemma 6.2.2 is satisfied, we can show that the stability properties of Theorem 6.2.2 are guaranteed for any bounded $\|H\|$.

Remark 6.2.3: The use of σ_i is found to be essential in obtaining sufficient conditions for boundedness in the presence of unmodeled interactions. In the absence of interactions, that is when each subsystem is decoupled, the design parameters $\sigma_i > 0$ cause an output error of $O(\sqrt{\sigma_i})$. This is a trade-off between boundedness of all signals in the presence of unmodeled interactions and the loss of exact convergence of the output error to zero in the absence of interactions.

6.3. Adaptive Control Using the Output of the Plant

We now consider the general problem of DAC of a large-scale system S which is described by (6.1) to (6.3). In this representation only local outputs are available for measurement and the output of each subsystem S_i is required to track the output y_{m_i} of an n_i-th order reference model M_i

$$\dot{x}_{m_i} = A_{m_i} x_{m_i} + b_{m_i} r_i \tag{6.40}$$

$$y_{m_i} = C^T_{m_i} x_{m_i} \tag{6.41}$$

whose transfer function $W_{m_i}(s)$

$$W_{m_i}(s) = C^T_{m_i}(sI-A_{m_i})^{-1} b_{m_i} = K_{m_i} \frac{Z_{m_i}(x)}{R_{m_i}(s)} \tag{6.42}$$

is chosen to be strictly positive real and $r_i(t)$ is a uniformly bounded reference input signal. In the transfer function

$$W_i(s) = h^T_i(sI-A_i)^{-1} b_i = K_i \frac{N_i(s)}{D_i(s)} \tag{6.43}$$

of the decoupled subsystem S_i it is assumed that $N_i(s)$ is a monic Hurwitz polynomial of degree n_i-1 *and* $D_i(s)$ *is a* monic polynomial of degree n_i. For ease of exposition we also assume that $K_i = K_{m_i} = 1$, $i=1,2,\ldots,N$.

The controller structure for subsystem S_i is

$$\begin{aligned} \dot{v}_i^{(1)} &= \Lambda_i v_i^{(1)} + g_i u_i \\ w_i^{(1)} &= C^T_i(t) v_i^{(1)} \end{aligned} \tag{6.44}$$

$$\begin{aligned} \dot{v}_i^{(2)} &= \Lambda_i v_i^{(2)} + g_i y_i \\ w_i^{(2)} &= d_{oi}(t) y_i + d^T_i(t) v_i^{(2)} \end{aligned} \tag{6.45}$$

where Λ_i is an $(n_i-1)\times(n_i-1)$ stable matrix and (Λ_i, g_i) is a controllable pair. The DAC input is given by

$$u_i = r_i + \theta^T_i w_i \tag{6.46}$$

where $\omega_i^T = [v_i^{(1)^T}, y_i, v_i^{(2)^T}]$ and $\theta_i^T = [c_i^T(t), d_{oi}(t), d_i^T(t)]$ is a (2n-1) dimensional adjustable parameter vector. It can be shown using the same procedure as in [7] that a constant vector θ_i^* exists such that for $\theta_i(t) = \theta_i^*$ the transfer function of the decoupled subsystem S_i given by (6.43) with controller (6.44) to (6.46) matches that of the reference model M_i given by (6.42).

The closed-loop subsystem S_i is described by the following equations

$$\begin{bmatrix} \dot{x}_i \\ \dot{v}_i^{(1)} \\ \dot{v}_i^{(2)} \end{bmatrix} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & \Lambda_i & 0 \\ g_i h_i^T & 0 & \Lambda_i \end{bmatrix} \begin{bmatrix} x_i \\ v_i^{(1)} \\ v_i^{(2)} \end{bmatrix} + \begin{bmatrix} b_i \\ g_i \\ 0 \end{bmatrix} (\theta_i^T \omega_i + r_i) + F_i \tag{6.47}$$

where

$$F_i = [f_i^T(x) \quad 0 \quad 0]^T. \tag{6.48}$$

Introducing θ_i^*, $Y_i^T = [x_i^T, v_i^{(1)^T}, v_i^{(2)^T}]$ and

$$A_{ci} = \left[\begin{array}{c|c|c} A_i + d_{oi}^* b_i h_i^T & b_i c_i^{*T} & b_i d_i^{*T} \\ \hline g_i d_{oi}^* h_i^T & \Lambda_i + g_i c_i^{*T} & g_i d_i^{*T} \\ \hline g_i h_i^T & 0 & \Lambda_i \end{array}\right], \quad b_{ci} = \left[\begin{array}{c} b_i \\ \hline g_i \\ \hline 0 \end{array}\right] \tag{6.49}$$

(6.47) is rewritten in the convenient compact form

$$\dot{Y}_i = A_{ci} Y_i + b_{ci}((\theta_i - \theta_i^*)^T \omega_i + r_i) + F_i. \tag{6.50}$$

For $\theta_i = \theta^*$ and $F_i = 0$ (6.50) is a non-minimal representation of the reference model M_i

$$\dot{x}_{ci} = A_{ci} x_{ci} + b_{ci} r_i, \quad x_{ci} = [x_{m_i}^T, v_{m_i}^{(1)^T}, v_{m_i}^{(2)^T}]^T. \tag{6.51}$$

The equations for the error $e_i = Y_i - x_{ci}$ can be expressed as

$$\dot{e}_i = A_{ci} e_i + b_{ci}(\theta_i - \theta_i^*)^T \omega_i + F_i \tag{6.52}$$

$$e_{oi} = h^T e_i = [1 \ \ 0 \ \ldots \ 0] e_i . \tag{6.53}$$

We now need to design an adaptive law for adjusting the parameter vector $\theta_i(t)$. For the decoupled subsystem S_i the adaptive law

$$\dot{\theta}_i = -\Gamma_i e_{oi} \omega_i \qquad \Gamma_i = \Gamma_i^T > 0 \tag{6.54}$$

guarantees that $e_i \to 0$ and $\theta_i \to$ constant vector as $t \to \infty$ for any uniformly bounded reference input $r_i(t)$. For the coupled subsystem S_i the error equation (6.52) does not have an equilibrium because of the forcing input F_i due to the interactions. The best we can do in this case is to guarantee boundedness for all signals in the closed-loop and convergence of the state errors to a residual set. We prove such a result for a modified adaptive law

$$\dot{\theta}_i = -\sigma_i \theta_i - \Gamma_i e_{oi} \omega_i \tag{6.55}$$

where σ_i is a positive scalar design parameter. The state error equation for the DAC scheme is described by

$$\dot{e} = A_c e + b_c(\theta - \theta^*)^T \omega + F(e + x_{mc}) \tag{6.56}$$

$$e_o = h_c^T e \tag{6.57}$$

where $e \triangleq [e_1^T, e_2^T \ \ldots \ e_N^T]^T$, $A_c \triangleq \text{diag}(A_{ci})$, $b_c \triangleq \text{diag}(b_{ci})$

$$\theta \triangleq \text{diag}(\theta_i), \quad \theta^* \triangleq \text{diag}(\theta_i^*), \quad x_{mc} \triangleq [x_{c1}^T, x_{c2}^T, \ldots, x_{cN}^T]$$

$$\omega \triangleq [\omega_1^T, \omega_2^T, \ldots, \omega_N^T]^T, \quad e_o \triangleq [e_{o1}, e_{o2}, \ldots, e_{oN}]^T, \quad h_c \triangleq \text{diag}(h)$$

and

$$F = \begin{bmatrix} 0 & \hat{A}_{12} & \hat{A}_{13} & \cdots & \hat{A}_{1N} \\ \hat{A}_{21} & 0 & & & \hat{A}_{2N} \\ \vdots & & \ddots & & \\ & & & \ddots & \\ \hat{A}_{N1} & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{6.58}$$

In (6.58)

$$\hat{A}_{ij} = \begin{bmatrix} A_{ij} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad i,j = 1,2,\ldots,N, \quad i \neq j.$$

The triple (A_c, b_c, h_c^T) satisfies the following equations

$$P_cA_c + A_c^TP_c = -Q_c - L_c \tag{6.59}$$

$$P_cb_c = h_c \tag{6.60}$$

where $P_c = \text{diag}(P_{ci})$, $Q_c = \text{diag}(q_iq_i^T)$, $L_c = \text{diag}(\varepsilon_iL_i)$, $P_{ci} = P_{ci}^T > 0$, $L_i = L_i^T > 0$, q_i is a vector and ε_i a positive scalar. Equations (6.59), (6.60) follow from the strictly positive realness of $W_{mi}(s) = h^T(sI - A_{ci})^{-1}b_{ci}$, $i = 1,2,\ldots,N$ and the Kalman-Yakubovich lemma. In view of (6.59), (6.60) the following theorem establishes the stability properties of the DAC scheme for the overall system S.

<u>Theorem 6.3.1</u>: Let

$$\lambda_c = \max_i \max \lambda(P_{ci}), \quad \lambda_\varepsilon = \min_i \min \lambda(\varepsilon_iL_i), \quad \lambda_i = \min(\Gamma_i^{-1}). \tag{6.61}$$

If

$$\lambda_\varepsilon > 2\lambda_c\|F\| \tag{6.62}$$

then all the signals in the closed loop system S are bounded. Furthermore

the solution $e(t)$, $\theta_i(t)$, $i=1,2,\ldots,N$ of (6.55) to (6.57) converges to the residual set

$$\mathcal{D} = \{e,\theta_i : \|\frac{e}{2}\|^2(\frac{\lambda_\varepsilon}{2} - \lambda_c\|F\|) + \sum_{i=1}^{N}\sigma_i\lambda_i(\|\theta_i - \theta_i^*\| - \frac{\|\Gamma_i^{-1}\theta_i^*\|}{2\lambda_i})^2 < d_c\} \tag{6.63}$$

where

$$d_c = \sum_{i=1}^{N}\sigma_i \frac{\|\Gamma_i^{-1}\theta_i^*\|^2}{4\lambda_i} + \frac{\lambda_c^2\|F\|^2}{(\frac{\lambda_\varepsilon}{2} - \lambda_c\|F\|)} \sup_t\|x_{mc}\|^2. \tag{6.64}$$

<u>Proof</u>: Choose the positive definite function

$$V = \frac{1}{2}\, e^T P_c e + \frac{1}{2}\sum_{i=1}^{N}(\theta_i - \theta_i^*)^T\Gamma_i^{-1}(\theta_i - \theta_i^*). \tag{6.65}$$

The time derivative of V along the solution of (6.55) to (6.57) is

$$\dot{V} = -\frac{1}{2}\, e^T(Q_c + L_c - (P_c F + F^T P_c))e + e^T P_c F x_{mc} - \sum_{i=1}^{N}\sigma_i(\theta_i - \theta_i^*)^T\Gamma_i^{-1}\theta_i \tag{6.66}$$

Using (6.61) we have

$$\dot{V} < -\|\frac{e}{2}\|^2(\frac{\lambda_\varepsilon}{2} - \lambda_c\|F\|) - \sum_{i=1}^{N}\sigma_i\lambda_i(\|\theta_i - \theta_i^*\| - \frac{\|\Gamma_i^{-1}\theta_i^*\|}{2\lambda_i})^2 + \sum_{i=1}^{N}\sigma_i \frac{\|\Gamma_i^{-1}\theta_i^*\|^2}{4\lambda_i} + \frac{\lambda_c^2\|F\|^2\|x_{mc}\|^2}{(\frac{\lambda_\varepsilon}{2} - \lambda_c\|F\|)}. \tag{6.67}$$

For $\lambda_\varepsilon > 2\lambda_c\|F\|$ $\dot{V} < 0$ outside the region $\mathcal{D}$ given by (6.63), (6.64). Hence the solution $\theta_i(t)$, $e(t)$ of (6.55) to (6.57) which starts outside $\mathcal{D}$ at $t=0$ will enter $\mathcal{D}$ in finite time. Once in $\mathcal{D}$ it cannot escape and will remain inside $\mathcal{D}$ for all $t \geq 0$. Since $e(t)$ and x_{mc} are bounded, Y and u are bounded and therefore all the signals in the closed loop are bounded.

Corollary 6.3.1: Assume $r_i = 0$, $x_{mi} = 0$, $i=1,2,\ldots,N$. If

$$\lambda_\varepsilon > 2\lambda_c \| F \| \tag{6.68}$$

and $\sigma_i = 0$ for $i = 1,2,\ldots,N$, then all the signals of the decentralized adaptive regulator scheme are bounded. Furthermore, $\|Y\| \to 0$, $\|\theta\| \to$ constant as $t \to \infty$.

Proof: For $x_{mc} = 0$ the DAC scheme described by (6.55) to (6.57) becomes a decentralized adaptive regulation problem with the objective to regulate the state Y to zero. Setting $\sigma_i = 0$ and $x_{mc} = 0$ in (6.55) to (6.57) we have

$$\dot{\theta}_i = -\Gamma_i y_i \omega_i \tag{6.68}$$

$$\dot{Y} = A_c Y + b_c (\theta - \theta^*)^T \omega + FY \tag{6.69}$$

$$Y_o = h_c^T Y, \qquad Y_o \triangleq [y_1, y_2, \ldots, y_N]^T. \tag{6.70}$$

Consider the positive definite function

$$V = \frac{Y^T P_c Y}{2} + \frac{1}{2} \sum_{i=1}^{N} (\theta_i - \theta_i^*)^T \Gamma_i^{-1} (\theta_i - \theta_i^*). \tag{6.71}$$

The time derivative of V along the solution of (6.68) to (6.70) is

$$\dot{V} = -\frac{1}{2} Y^T (Q_c + L_c - (P_c F + F^T P_c)) Y. \tag{6.72}$$

Using (6.61) we have

$$\dot{V} < -\frac{\|Y\|^2}{2} (\lambda_\varepsilon - 2\lambda_c \| F \|). \tag{6.73}$$

For $\lambda_\varepsilon > 2\lambda_c \| F \|$ $\dot{V} < 0$ and therefore Y, θ_i $i = 1,2,\ldots,N$ are uniformly bounded. Since $\ddot{V}$ is uniformly bounded, $\dot{V}$ is uniformly continuous. Hence $\lim_{t\to\infty} \dot{V} = 0$, i.e. $\|Y\| \to 0$ and $\|\theta\| \to$ constant as $t \to \infty$.

6.4. Discussion and Example

In this section the problem of DAC of a class of large-scale systems has been examined. We have obtained sufficient conditions for adaptive regulation and tracking using DACRs. The proposed approach can insure exact convergence to the equilibrium point in the case of regulation and boundedness and convergence to a residual set in the case of tracking. The size of the residual set depends on design parameters, the characteristics of the reference model and reference input signal, and the size of the interconnection matrix. We demonstrate the effectiveness of the proposed schemes by digital simulation of a DAC scheme for the second order system

$$\dot{x}_1 = 5x_1 + a_{12}x_2 + u_1 \tag{6.74}$$

$$\dot{x}_2 = 3x_2 + a_{21}x_1 + u_2. \tag{6.75}$$

In this case we have two first order subsystems and it is required to design DACRs u_1 and u_2 such that the states x_1, x_2 are regulated to zero or track the corresponding states x_{m1}, x_{m2} of the reference model

$$\dot{x}_{m1} = -4x_{m1} + r_1 \qquad (M_1) \tag{6.76}$$

and

$$\dot{x}_{m2} = -5x_{m2} + r_2 \qquad (M_2) \tag{6.77}$$

respectively. The interconnection matrix H is given by $H = \begin{bmatrix} 0 & a_{12} \\ a_{22} & 0 \end{bmatrix}$.

Consider the decoupled subsystems

$$\dot{x}_1 = 5x_1 + u_1 \qquad (S_1) \tag{6.78}$$

$$\dot{x}_2 = 3x_2 + u_2 \qquad (S_2). \tag{6.79}$$

a. Regulation: Following the procedure of Section 6.2, we choose for S_1

$$u_1 = -K_1x_1, \qquad \dot{K}_1 = 5x_1^2 \tag{6.81}$$

and for S_2

$$u_2 = -K_2x_2, \qquad \dot{K}_2 = 5x_2^2. \tag{6.82}$$

Since condition (6.25) is satisfied for any $\varepsilon > 0$ we would expect $x_1, x_2 \to 0$ as $t \to \infty$ for any bounded $\|H\|$.

In Fig. 6.1a,b the adaptive regulation of x_1, x_2 to zero is shown for $a_{12} = 4$ and $a_{21} = 3$. In Fig. 6.1c,d we show the time response of the controller parameters K_1, K_2 which converge to a constant. By increasing the size of the interconnections to $a_{12}=10$, $a_{21} = 15$ the states x_1, x_2 are still regulated to zero as shown in Fig. 6.2a,b. As a result of the stronger interactions the controller parameters K_1, K_2 converge to constants which are higher than those of Fig. 6.1c,d.

b. Tracking: For tracking we use for S_1,

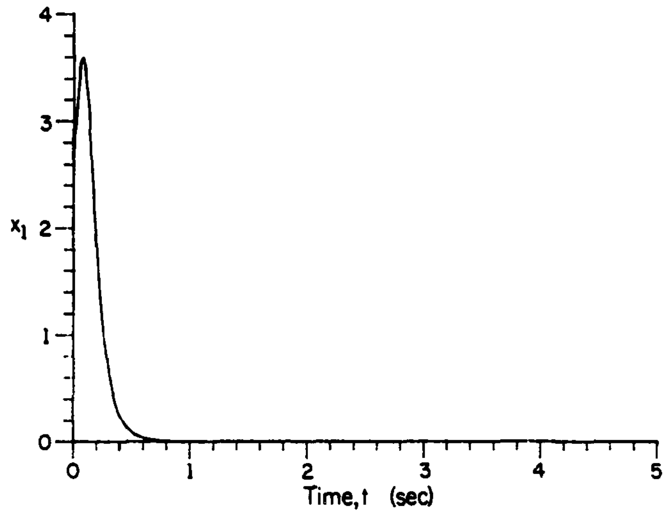
$$u_1 = -K_1x_1 + r_1, \qquad \dot{K}_1 = 5e_1x_1 - \sigma_1K_1 \tag{6.83}$$

and for S_2

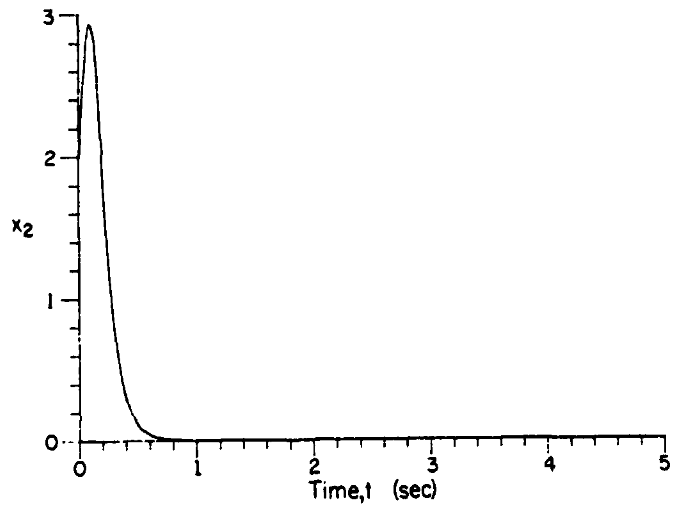
$$u_2 = -K_2x_2 + r_2, \qquad \dot{K}_2 = 5e_2x_2 - \sigma_2K_2 \tag{6.84}$$

where $e_1 \triangleq x_1 - x_{m1}$, $e_2 \triangleq x_2 - x_{m2}$. In Fig. 6.3a,b,c,d we show the time responses of the state errors e_1, e_2 and controller parameters K_1, K_2 for $a_{12} = 4$, $a_{21} = 3$, $r_1 = 2\sin t$, $\sigma_1 = 0.01$, $r_2 = 3\sin 2t$ and $\sigma_2 = 0.02$. Both state errors converge to a bounded residual set and are close to zero. By increasing the interconnections to $a_{12} = 10$, $a_{21} = 9$ and leaving all the other variables the same as in Fig. 6.3 the state errors still converge to a residual set

and the controller parameters remain bounded as shown in Fig. 6.4a,b,c,d. The invariance of the stability properties of DACRs (6.81) to (6.84) to the size of the interactions is due to condition (6.25) of Lemma 6.2.2, which holds for the system (6.74), (6.75). For this example the DACR pushes the controller gains in a direction so that the interactions are dominated. In general, however, this is not true since condition (6.25) cannot be satisfied for all systems of the class considered in this section. When an upper bound for the norm of the interactions is known the DACRs for regulation or tracking can be designed so that the sufficient conditions for stability and boundedness obtained in this section are satisfied. This can be achieved by proper selection of the adaptive gains and reference model.

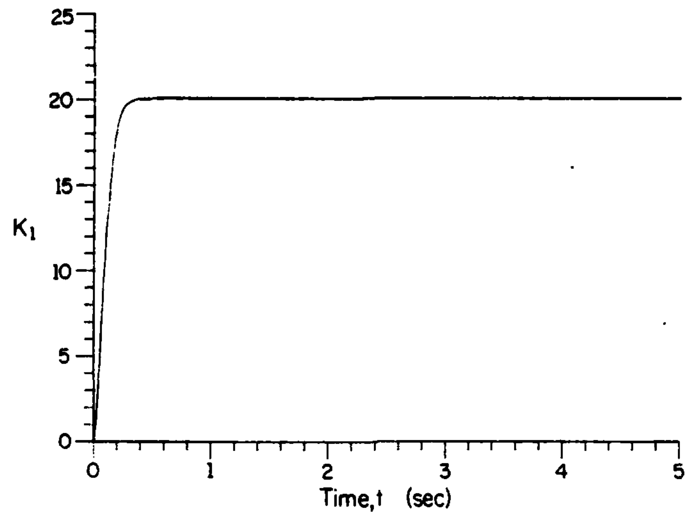


(a)

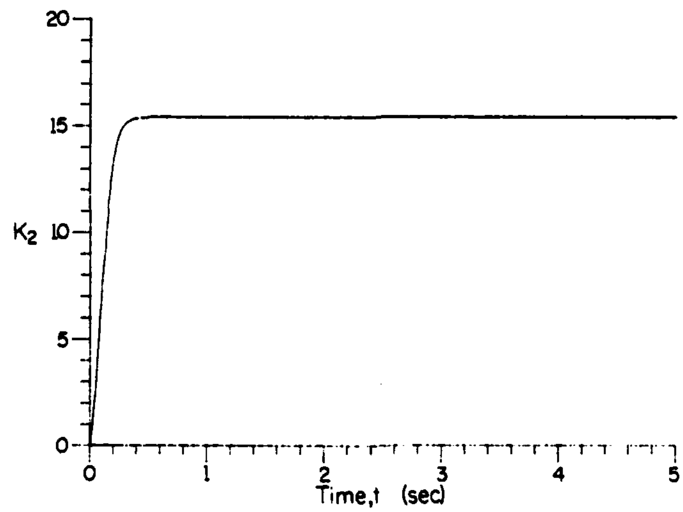


(b)

Fig. 6.1. Adaptive regulation results for $a_{12} = 4$, $a_{21} = 3$.



(c)



(d)

Fig. 6.1. Adaptive regulation results for $a_{12} = 4$, $a_{21} = 3$.

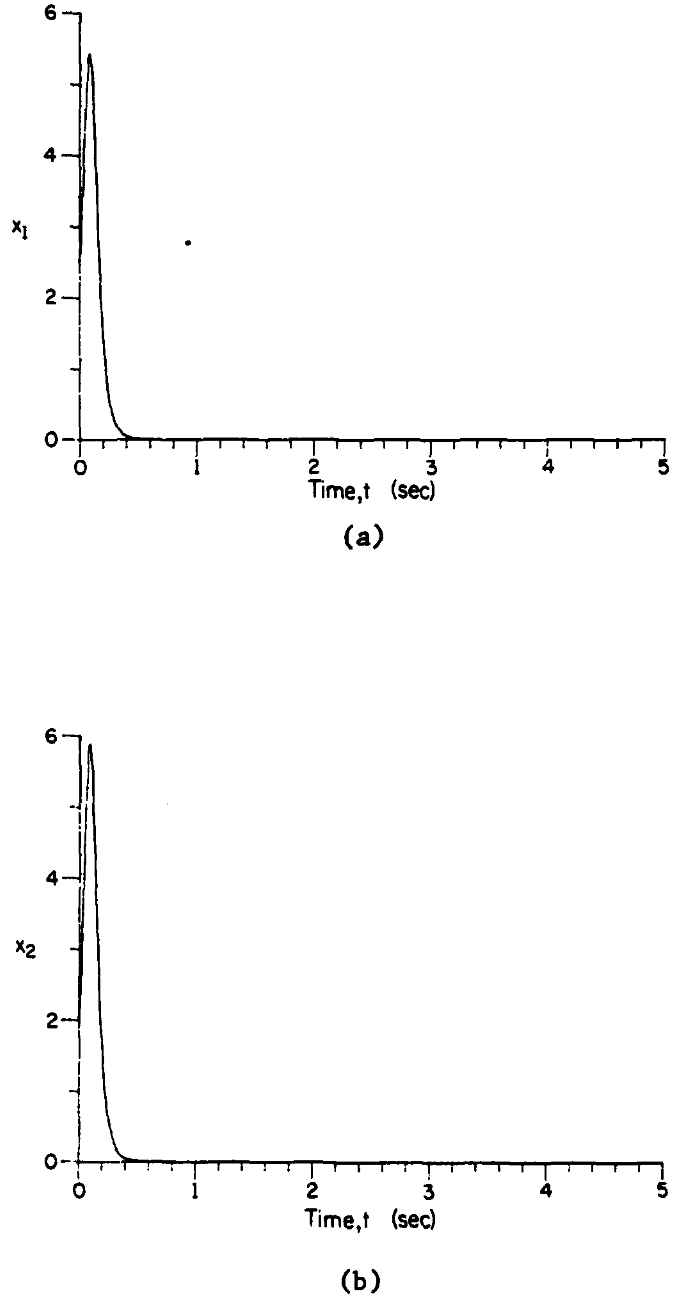
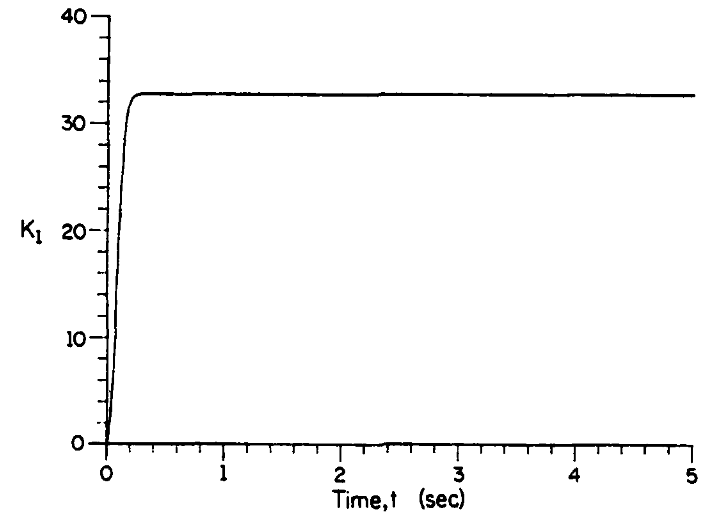
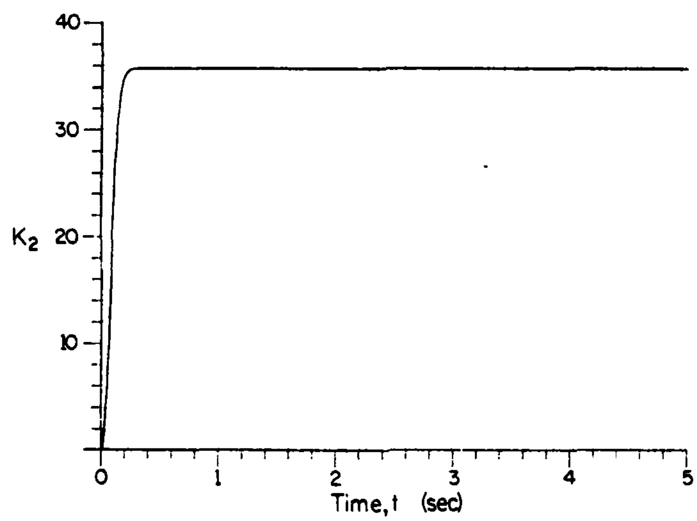


Fig. 6.2. Adaptive regulation results for $a_{12} = 10$, $a_{21} = 15$.

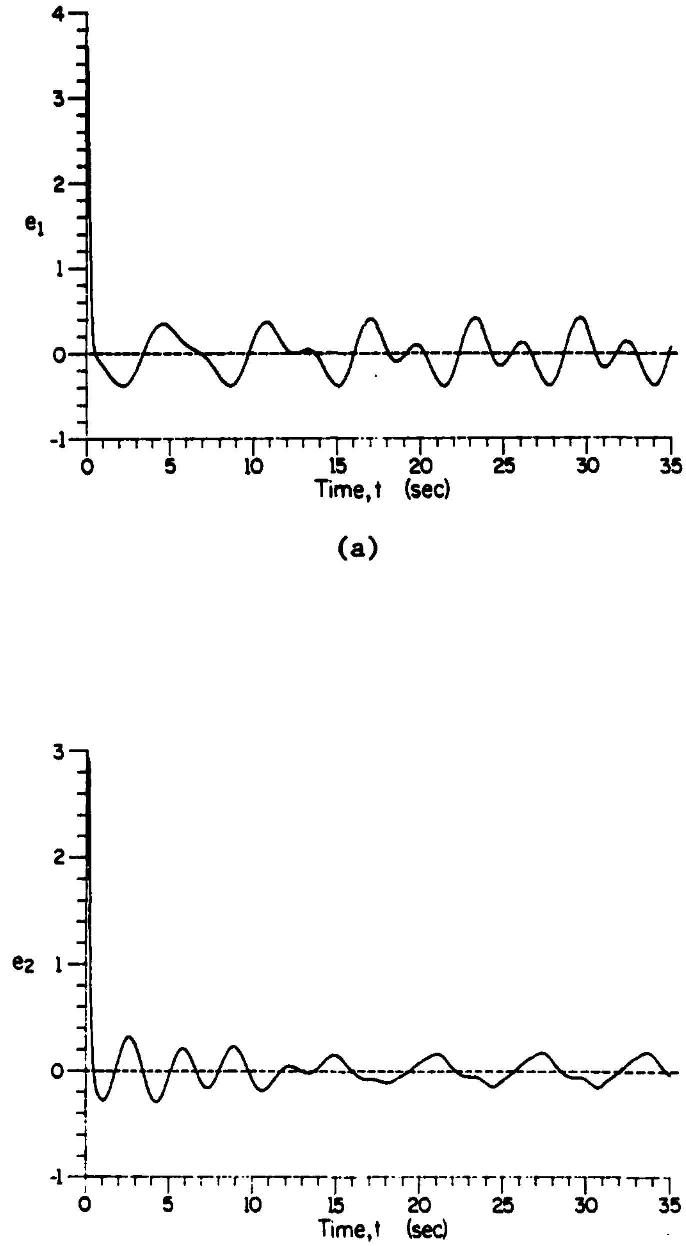


(c)



(d)

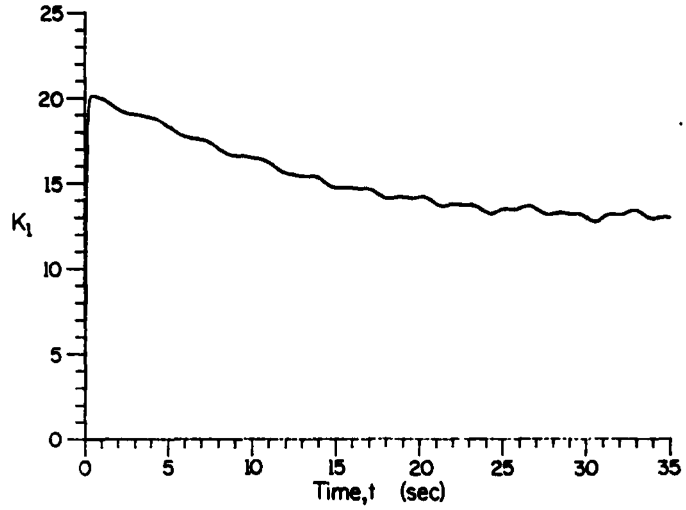
Fig. 6.2. Adaptive regulation results for $a_{12} = 10$, $a_{21} = 15$.



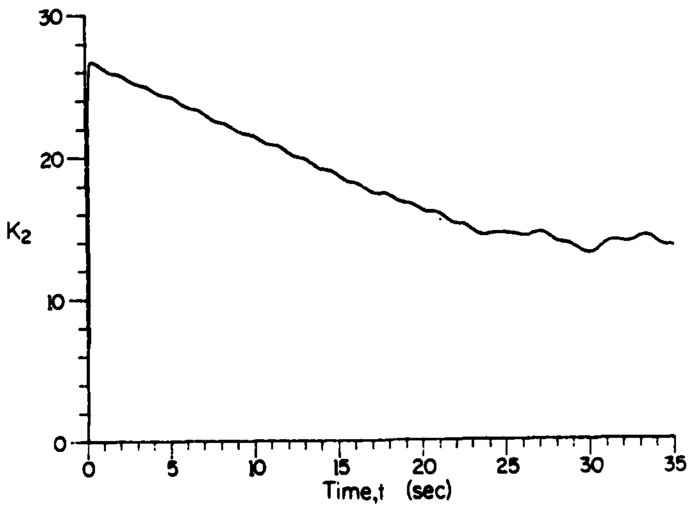
(a)

(b)

Fig. 6.3. Tracking results for $a_{12} = 4$, $a_{21} = 3$.

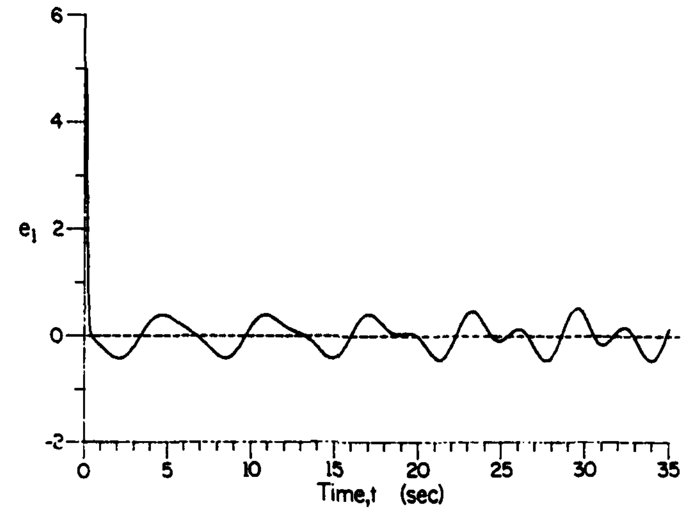


(c)

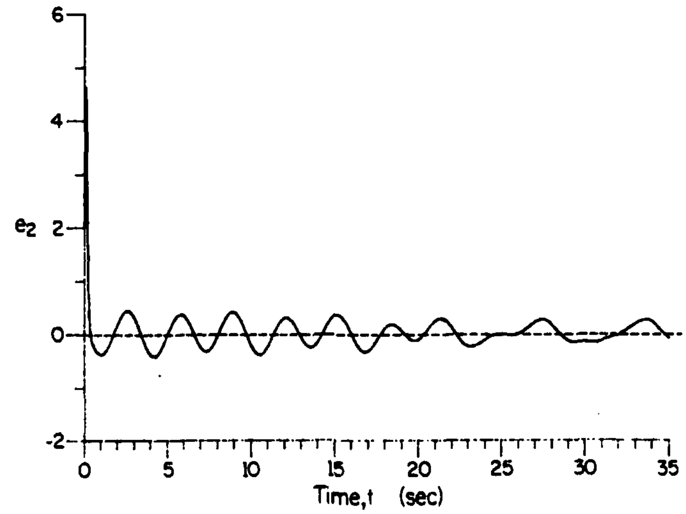


(d)

Fig. 6.3. Tracking results for $a_{12} = 4$, $a_{21} = 3$.



(a)



(b)

Fig. 6.4. Tracking results for $a_{12} = 10$, $a_{21} = 9$.

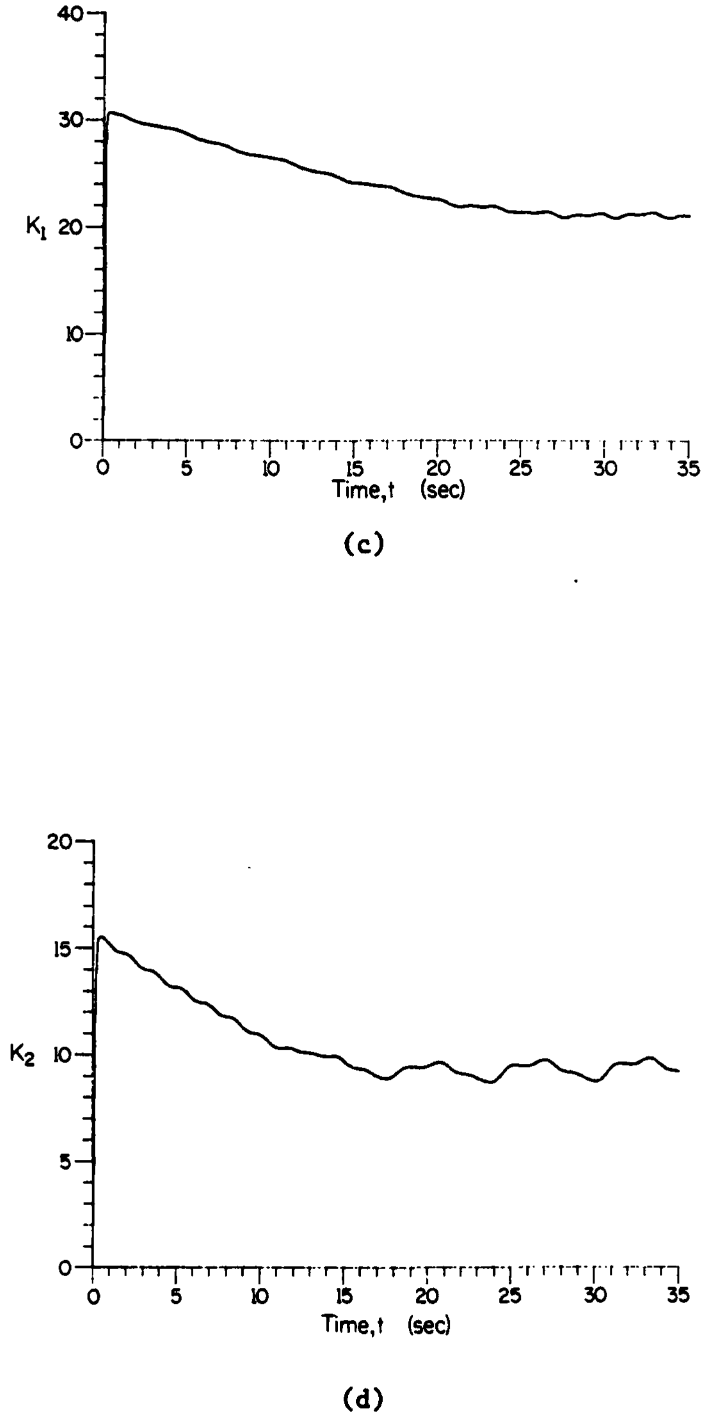


Fig. 6.4. Tracking results for $a_{12} = 10$, $a_{21} = 9$.

7. CONCLUSIONS

In this thesis we analyze the robustness properties of model reference adaptive schemes with respect to modeling errors. In Sections 2 to 5 the modeling error consists of fast parasitics whereas in Section 6 is due to neglected interactions between subsystems. In the case of identifiers and adaptive observers we introduce the notion of dominantly rich inputs, that is inputs which are rich for the dominant modes, but do not contain high frequencies in the parasitic range, in order to establish robustness with respect to parasitics. Detailed bounds for the composite state/parameter error are established which indicate possibilities for reducing the error by a proper choice of the input signal. We show that the assumption of weak observability of parasitics in the plant output is crucial for robustness. When parasitics are strongly observable we show that identifiers and adaptive observers are no longer robust. In this case robustness is established by properly modifying the adaptive algorithms or filtering the plant output.

The effects of unmodeled fast dynamics on the stability and performance of adaptive control schemes are analyzed. We show that in the regulation problem global stability properties are no longer guaranteed, but a region of attraction exists for exact adaptive regulation. In the case of tracking we proposed a more robust adaptive law which guarantees the existence of a region of attraction from which all signals converge to a residual set which contains the equilibrium for exact tracking.

The adaptive control of a class of large-scale systems is also considered. Decentralized adaptive controllers are proposed for adaptive regulation and tracking. We obtain sufficient conditions in the form of

algebraic criteria which can guarantee stability or boundedness under certain structural perturbations. The theoretical results obtained are illustrated by simulations of numerical examples.

There are several possible directions for future research. The results obtained in this thesis for deterministic algorithms can be extended to algorithms in a stochastic environment. A wider class of algorithms can be analyzed with respect to more general structural perturbations following the same procedure as in this thesis. The stability properties of indirect adaptive controllers such as self-tuning regulators with respect to modeling errors such as fast parasitics can be examined. The decentralized adaptive identification or control of a wider class of large-scale systems can be analyzed.

APPENDIX A. ERROR SYSTEM OF MINIMAL-FORM ADAPTIVE OBSERVERS

Case 1: Adaptive observer [33]

Without loss of generality let us assume that the model of the dominant part of the plant (2.4) is in the observable canonical form

$$\dot{x} = \left[\begin{array}{c|c} & I \\ -\alpha & --- \\ & 0 \end{array}\right] x + bu + H\eta \tag{A.1}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1}B_f\dot{u} \tag{A.2}$$

$$y = c^Tx = [1 \quad 0 \ \ldots \ 0]x = x_1. \tag{A.3}$$

The algorithm [33] for the nth order adaptive observer based on the dominant part (A.1), (A.3) without the parasitics ($\eta = 0$ in (A.1)) is given by the equations (A.4) through (A.11), below. The observer equation is

$$\dot{z} = Kz + [k-\hat{a}(t)]y + \hat{b}(t)u + w + r \tag{A.4}$$

$$\hat{y} = c^Tz = z_1 \tag{A.5}$$

where w and r are auxiliary signals formed by the output error $e_1 \triangleq \hat{y}-y$ and the components

$$v_i = \frac{s^{n-i}}{s^{n-1}+d_2s^{n-2}+\cdots+d_n}x_1, \quad q_i = \frac{s^{n-i}}{s^{n-1}+d_2s^{n-2}+\cdots+d_n}u \tag{A.6}$$

of the vectors v and q as follows

$$w = -e_1 \begin{bmatrix} 0 \\ v^T \Gamma A_2 v \\ \vdots \\ v^T \Gamma A_j v \\ \vdots \\ v^T \Gamma A_n v \end{bmatrix}, \quad r = -e_1 \begin{bmatrix} 0 \\ q^T M A_2 q \\ \vdots \\ q^T M A_j q \\ \vdots \\ q^T M A_n q \end{bmatrix}. \qquad \text{(A.7)}$$

Matrices A_j are

$$A_j = \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{16em}}^{n-j+1} & \overbrace{\hspace{10em}}^{j-2} \end{array} \\ \left[\begin{array}{ccccccccccccc} 0 & -d_j & -d_{j+1} & \cdot & \cdot & \cdot & -d_n & & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -d_j & \cdot & \cdot & \cdot & -d_{n-1} & & -d_n & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & -d_j & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & -d_n \\ 0 & 1 & d_2 & \cdot & \cdot & \cdot & d_{j-1} & & 0 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & 1 & d_2 & \cdot & \cdot & d_{j-2} & & -d_{j-1} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & 1 & d_2 & & d_3 & \cdot & \cdot & \cdot & -d_{j-1} \end{array}\right] \end{array} \begin{array}{l} \\ \left.\vphantom{\begin{array}{c} a \\ a \\ a \\ a \end{array}}\right\} j-1 \\ \left.\vphantom{\begin{array}{c} a \\ a \\ a \\ a \end{array}}\right\} n-j+1 \end{array} \qquad \text{(A.8)}$$

and $\Gamma = \Gamma^T > 0$, $M = M^T > 0$ while

$$K = \left[\begin{array}{c|c} k & \begin{array}{c} I \\ \hline 0 \end{array} \end{array}\right], \quad d = \begin{bmatrix} 1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \qquad \text{(A.9)}$$

are chosen such that $c^T(sI-K)^{-1}d$ is positive real. The adaptive laws for updating the estimated parameters are given by

$$\dot{\phi} = -\Gamma e_1 u = -\dot{\hat{a}}(t) \tag{A.10}$$

$$\dot{\Psi} = -Me_1 q = \dot{\hat{b}}(t) \tag{A.11}$$

where $\phi \overset{\Delta}{=} \alpha - \hat{a}(t)$ and $\Psi \overset{\Delta}{=} \hat{b}(t) - B$ are the parameter errors.

<u>Case 2</u>: Adaptive observer [32], [33]

The following "modal" canonical form is chosen for the dominant part of the plant (2.4)

$$\dot{x} = \left[\begin{array}{c|c} a & \begin{array}{c} h^T \\ \hline \Lambda \end{array} \end{array}\right] x + bu + H\eta \tag{A.12}$$

$$\mu\dot{\eta} = A_f \eta + \mu A_f^{-1} B_f \dot{u} \tag{A.13}$$

$$y = c^T x = x_1 \tag{A.14}$$

where $h^T = [1 \;\; 1 \; \dots \; 1]$, Λ is an $(n-1)\times(n-1)$ diagonal matrix with arbitrary but known constant and negative diagonal elements $-\lambda_i$ $(i=2,\dots,n)$ and a, B are the unknown vectors to be identified. It is shown in [5] that any completely observable system can be represented in this "modal" canonical form. The structure of the adaptive observer based on (A.2) with $\eta = 0$ is summarized in the equations (A.15) through (A.20). The adaptive observer equation is

$$\dot{z} = Kz + (k - \hat{a}(t))y + \hat{b}(t)u + w + r \tag{A.15}$$

$$\hat{y} = c^T z = z_1 \tag{A.16}$$

where w and r are auxiliary signals formed by the derivatives of the parameter error components and the components

$$v_i = \frac{1}{s+\lambda_i} x_i, \quad q_i = \frac{1}{s+\lambda_i} u \quad (i=2,\ldots,n) \tag{A.17}$$

of the vectors v and q as follows

$$w = -\begin{bmatrix} 0 \\ \dot{\phi}_2 v \\ \vdots \\ \dot{\phi}_n v \end{bmatrix}, \quad r = \begin{bmatrix} 0 \\ \dot{\Psi}_2 q \\ \vdots \\ \dot{\Psi}_n q \end{bmatrix}. \tag{A.18}$$

Moreover, $K\left[\begin{array}{c|c} -\lambda_1 & h^T \\ \hline 0 & \\ \vdots & \Lambda \\ 0 & \end{array}\right]$ is stable, the transfer function $c^T(sI-K)^{-1}d = \frac{1}{s+\lambda_1}$ is strictly positive real, and $d = (1\ 0\ \ldots\ 0)^T$. Note that the first components of the vectors v and q are $v_1 = x_1$, $q_1 = u$, respectively. The adaptive laws for adjusting the parameters are

$$\dot{\phi} = -\Gamma e_1 u = \dot{\hat{a}}(t) \tag{A.19}$$

$$\dot{\Psi} = -Me_1 q = \dot{\hat{b}}(t). \tag{A.20}$$

The stability properties of the adaptive observer in the presence of parasitics for Cases 1 and 2 are desc:ibed by the following error equations

$$\dot{e} = Ke + \phi x_1 + \Psi u - H\eta + w + r, \quad e_1 = c\ e \tag{A.21}$$

$$\dot{\phi} = \Gamma e_1 u \tag{A.22}$$

$$\dot{\Psi} = -Me_1 q. \tag{A.23}$$

<u>Proposition</u>: For some vector signals v, q, w and r with $v = G(p)x_1$, $q = G(p)u$, $w = w(\dot{\phi},u)$ and $r = r(\dot{\Psi},q)$ the system (A.21) is input $([x_1,u,\eta^T]^T)$-output (e_1)

equivalent with the system (A.24) provided (c^T, K) is completely observable

$$\dot{\varepsilon} = K\varepsilon + d[\phi^T v + \Psi^T q] - H\eta, \quad \varepsilon_1 = c^T\varepsilon = e_1 \tag{A.24}$$

<u>Proof</u>: From (A.21), (A.24)

$$c^T(pI-K)^{-1}[\phi x_1 + \Psi u + w + r - d(\phi^T v + \Psi^T q)] \tag{A.25}$$

where 'p' is the d/dt operator. From (A.25) we have

$$\sum_{i=1}^{n} p^{n-i}[\phi_i x_1 + \Psi_i u + w_i + r_i - d_i\phi^T v - d_i\Psi^T q] = 0 \tag{A.26}$$

where i denotes the ith element of the corresponding vector and (A.26) is satisfied by choosing v, q, w, and r as given in Case 1 and Case 2. By considering the two equivalent systems (A.21) to (A.23) and (A.24), (A.22), (A.23), it is obvious that boundedness of $[\varepsilon^T, \phi^T, \Psi^T]^T$ will imply boundedness of $[e^T, \phi^T, \Psi^T]$. Thus the stability of the adaptive observers in Cases 1 and 2 in the presence of parasitics is equivalent to the stability of the following differential equations

$$\dot{\varepsilon} = K\varepsilon + d[\phi^T v + \Psi^T q] - H\eta \tag{A.27}$$

$$\varepsilon_1 = e_1 \tag{A.28}$$

$$\dot{\phi} = -\Gamma\varepsilon_1 v \tag{A.29}$$

$$\dot{\Psi} = -M\varepsilon_1 q \tag{A.30}$$

where K, d, v, and q are defined differently in Cases 1 and 2.

APPENDIX B. ERROR SYSTEM OF A NONMINIMAL FORM ADAPTIVE OBSERVER

From (2.7) and (2.49)

$$\frac{y(s)}{u(s)} = G(s) = G_p(s) + c^T(sI-A)^{-1}H\,\frac{\eta(s)}{u(s)} \tag{B.1}$$

where

$$G_p(s) = c^T(sI-A)^{-1}B \tag{B.2}$$

is the transfer function of the plant when $H\eta = 0$. Let

$$G_p(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}{s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n} \tag{B.3}$$

where

$$\alpha \triangleq [\alpha_1,\alpha_2,\ldots,\alpha_n]^T \quad \text{and} \quad B \triangleq [b_1,b_2,\ldots,b_n]^T \tag{B.4}$$

are the unknown parameter vectors.

Consider a polynomial $\prod_{i=2}^{n}(s+\lambda_i)$ which is relatively prime to the numerator as well as the denominator polynomials of $G_p(s)$ and $\lambda_i \neq \lambda_j$ for $i, j = 2,3,\ldots,n$. Dividing the numerator and denominator of $G(s)$ by $\prod_{i=2}^{n}(s+\lambda_i)$ and expanding them into partial fractions we have

$$G(s) = \frac{b_1 + \frac{b_2}{s+\lambda_2} + \cdots + \frac{b_n}{s+\lambda_n}}{s - a_1 - \frac{a_2}{s+\lambda_2} - \cdots - \frac{a_n}{s+\lambda_n}} + \frac{\frac{c^T \operatorname{adj}(sI-A)^{-1}H}{\prod_{i=2}^{n}(s+\lambda_i)}\;\frac{\eta(s)}{u(s)}}{s - a_1 - \frac{a_2}{s+\lambda_2} \cdots \frac{a_n}{s+\lambda_n}}. \tag{B.5}$$

Note that

$$\frac{c^T \operatorname{adj}(sI-A)^{-1}}{\prod_{i=2}^{n}(s+\lambda_i)} = \frac{[s^{n-1},s^{n-2},\ldots,1]}{\prod_{i=2}^{n}(s+\lambda_i)} = \Big[\sum_{i=2}^{n}\frac{t_{1i}}{s+\lambda_i},\ \sum_{i=2}^{n}\frac{t_{2i}}{s+\lambda_i},\ \ldots,$$

$$\sum_{i=2}^{n}\frac{t_{ni}}{s+\lambda_i}\Big] \tag{B.6}$$

(B.5) can be written as

$$y(s) = \frac{1}{s}[b_1u(s) + a_1y(s) + \sum_{i=2}^{n}\frac{[b_iu(s) + a_iy(s) + T_iH\eta(s)]}{s + \lambda_i} \tag{B.7}$$

where

$$T_i = [t_{1i}, t_{2i},\ldots,t_{ni}] \tag{B.8}$$

(B.7) is represented in a block diagonal form in Figure B.1. The block diagram of Figure B.1 contains (3n-2) integrators and is a nonminimal realization of the dominant part of the plant.

The term $h^T\exp(\Lambda t)\bar{x}(0)$ in Figure B.1 is added so that Figure B.1 is equivalent to the corresponding figure of a minimal realization of (B.7) including initial conditions. Here $\bar{x} = [x_2,x_3,\ldots,x_n]$ and x is the state of the minimal realization based on (B.7).

A nonminimal state-space representation can be obtained from Figure B.1 by defining

$$R_s = [r_2,r_3,\ldots,r_n]^T, \quad w_s = [w_2,w_3,\ldots,w_n]^T$$
$$Z_s = [z_2,z_3,\ldots,z_n]^T \text{ and } T_s = [T_2',T_3',\ldots,T_n']^T \tag{B.9}$$

i.e.

$$\begin{bmatrix} \dot{y} \\ \dot{Z}_s \\ \dot{W}_s \\ \dot{R}_s \end{bmatrix} = \begin{bmatrix} a_1 & a_s^T & b_s^T & h^T \\ h & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} y \\ Z_s \\ W_s \\ R_s \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ h \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_s \end{bmatrix} H\eta + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} h^T \ell^{\Lambda t}\bar{x}(0) \tag{B.10}$$

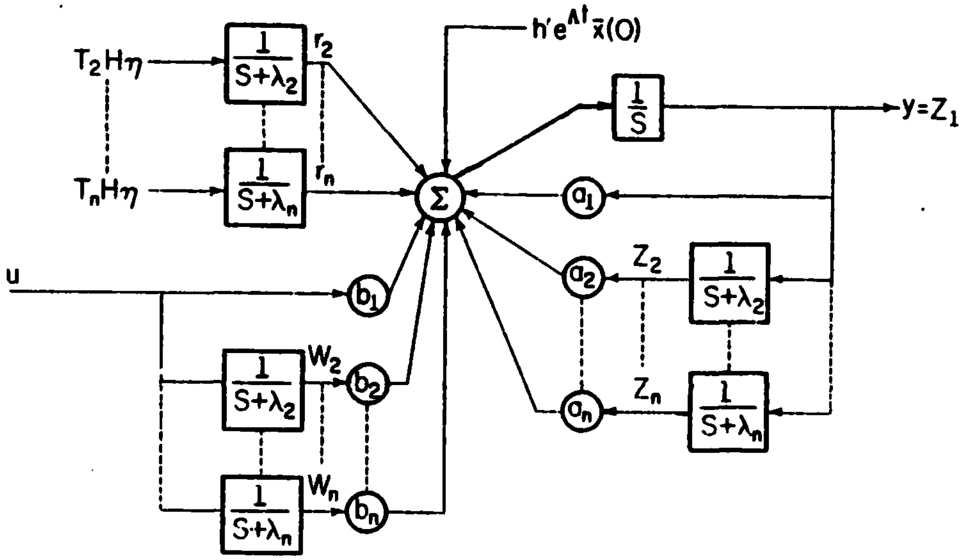


Fig. B.1. Nonminimal representation of the dominant part of the plant.

$$\bar{y} = [1 \quad 0 \; \dots \; 0] \begin{bmatrix} y \\ Z_s \\ W_s \\ R_s \end{bmatrix} \tag{B.11}$$

$$\mu\dot{\eta} = A_f\eta + \mu A_f^{-1}B_f\dot{u} \tag{B.12}$$

where $h^T = [1 \quad 1 \quad \dots \quad 1]$ and $\Lambda = \mathrm{diag}(\lambda_i)$. The structure of the adaptive observer for (B.10), (B.11) in the absence of parasitics (i.e. $\eta = 0$, $R_s = 0$) is given in [32], [33] and the basic equations are reviewed below. The observer equations are

$$\dot{\hat{y}} = \hat{a}_1(t)\hat{y} + a_s^T\hat{Z}_s + \hat{b}_1(t)u + b^T(t)W_s - \lambda_1(\hat{y}-y) \tag{B.13}$$

$$\dot{\hat{Z}}_s = \Lambda\hat{Z}_s + hy \tag{B.14}$$

$$\dot{\hat{W}}_s = \Lambda\hat{W}_s + hu \tag{B.15}$$

where $\hat{y}(0) = 0$, $\hat{Z}_s(0) = 0$, $\hat{W}_s(0) = 0$. The adaptive laws for adjusting the unknown parameters are given by

$$\dot{\phi} = -\Gamma e_1 v \tag{B.16}$$

$$\dot{\Psi} = -Me_1 p \tag{B.17}$$

where $\phi \triangleq [(\hat{a}_1(t)-a_1),(\hat{a}(t)-a_s)^T]$, $\Psi \triangleq [(\hat{b}_1(t)-b_1),\ (\hat{b}(t)-b_s)^T]$ and $v = [y,\hat{Z}_s^T]^T$, $p = [u,\hat{W}_s^T]^T$. The stability properties of the nonminimal adaptive observer (B.13) to (B.17) in the presence of parasitics is described by

$$\dot{e}_1 = -\lambda_1 e_1 + d[v^T\phi + q^T\Psi] - h^T R_s - h^T \ell^{\Lambda t} \bar{x}(0) \tag{B.18}$$

$$\dot{\phi} = -\Gamma e_1 v \tag{B.19}$$

$$\dot{\Psi} = -M e_1 p \tag{B.20}$$

where (B.18) is obtained by substracting (B.13) from (B.11).

APPENDIX C. ERROR SYSTEM OF A PARAMETERIZED ADAPTIVE OBSERVER

Equation (2.4) can be represented as

$$\dot{x} = Fx + gy + Bu + H\eta \tag{C.1}$$

where

$$F = \begin{bmatrix} -f_1 & 1 & 0 \dots & 0 \\ -f_2 & 0 & 1 & \\ \vdots & \vdots & & 1 \\ -f_n & 0 & 0 \dots & 0 \end{bmatrix}, \quad g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \tag{C.2}$$

and g satisfies $gc^T = A-F$.

From (C.1)

$$x = \ell^{Ft}x_o + \int_o^t \ell^{F(t-\tau)}[gy(\tau) + Bu(\tau)]d\tau + \int_o^t \ell^{F(t-\tau)}H\eta(\tau)d\tau. \tag{C.3}$$

The first convolution integral can be reduced to

$$\int_o^t \ell^{F(t-\tau)}[Iy(\tau),\ Iu(\tau)]d\tau \cdot p^* = M(t)p^* \tag{C.4}$$

where $p^{*T} = [g^T, B^T]$. By taking

$$D(t) = \int_o^t \ell^{F(t-\tau)}H\eta(\tau)d\tau \tag{C.5}$$

we have

$$\dot{M}(t) = FM(t) + [Iy, Iu] \qquad M(0) = 0 \tag{C.6}$$

$$X(t) = M(t)p^* + EXP(Ft)X(0) + D(t) \tag{C.7}$$

$$\dot{D}(t) = FD(t) + H\eta(t) \qquad D(0) = 0 \tag{C.8}$$

$$\mu\dot{\eta} = A_f\eta(t) + \mu A_f^{-1}B_f\dot{u} \tag{C.9}$$

$$y = c^Tx = x_1 \tag{C.10}$$

where p^* contains all the unknown parameters of the matrix A and vector B in (2.7). The adaptive observer [34] for observing the state x and estimating p^* when $D(t) = 0$ is reviewed below. The observer equation is

$$\hat{x}(t) = M(t)\hat{p}(t) + EXP(Ft)\hat{x}(0) \tag{C.11}$$

$$\hat{y}(t) = c^T\hat{x}. \tag{C.12}$$

The adaptive law for updating the unknown vector $\hat{p}$, obtained by minimizing

$$\zeta_1 = \frac{1}{2} e_1^2 \tag{C.13}$$

is given by [34]

$$\dot{\hat{p}}(t) = -GM^T(t)ce_1 \tag{C.14}$$

where $G = G^T > 0$ and $e_1 \triangleq \hat{y}-y.$

From (C.7), (C.11), and (C.14) the state error $e \triangleq x-\hat{x}$ and parameter error $\Delta p \triangleq p^*-\hat{p}$ satisfy

$$\dot{e} = -M(t)GM^T(t)cc^Te + [FM(t)+(Iy,Iu)]\Delta p + \exp(Ft)Fe(0)-FD(t)-H\eta(t) \tag{C.15}$$

$$\Delta\dot{p} = -GM^T(t)cc^Te. \tag{C.16}$$

APPENDIX D. PARALLEL IDENTIFIER ERROR SYSTEM

This appendix is devoted to the derivation of (3.26) to (3.28) from (3.12) and (3.14) to (3.20). Substracting (3.14) from (3.12) yields

$$y(k) - y_p(k) = \sum_{i=1}^{n} a_i[y(k-i)-y_p(k-i)] + \sum_{i=1}^{n}[a_i-\hat{a}_i(k)]y_p(k-i)$$

$$+ \sum_{i=1}^{n}[b_i-\hat{b}_i(k)]u(k-i) + y_u(k). \tag{D.1}$$

Given (3.24), (3.29), and (3.30), (D.1) can be rewritten as

$$e(k+1) = Ae(k) + bw(k) \tag{D.2}$$

where

$$w(k) = \sum_{i=1}^{n}\{[a_i-\hat{a}_i(k+1)]y_s(k-i+1) + [b_i-\hat{b}_i(k+1)]u(k-i+1)\} + y_u(k+1). \tag{D.3}$$

Given (3.16) and (3.25), (D.3) can be rewritten as

$$w(k) = \theta_p^T(k)\Delta p_p(k+1) + y_u(k+1). \tag{D.4}$$

Subtracting both sides of (3.16) from the desired parameter vector yields

$$\Delta p_p(k+1) = p_p(k) - \frac{F_p(k)\theta_p(k)\nu_p^o(k+1)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}. \tag{D.5}$$

From (3.14) and (3.18)

$$\hat{p}_p^T(k-1)\theta_p(k-1) = y_p(k) - \frac{\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)\nu_p^o(k)}{1+\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)}. \tag{D.6}$$

Substituting (D.6) into (3.19) yields

$$\nu_p^o(k) = [1+\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)]\{y(k)-y_p(k) + \sum_{i=1}^{n} c_i[y(k-i)-y_p(k-i)]\}. \tag{D.7}$$

Using (D.1), (3.24), (3.28) and (3.29) converts (D.7) to

$$\nu_p^o(k) = [1+\theta_p^T(k-1)F_p(k-1)\theta_p(k-1)][(a+c)e_p(k-1)+w(k-1)]. \tag{D.8}$$

Combining (D.4) and (D.5) yields

$$w(k) = \theta_p^T(k)\Delta p_p(k) - \frac{\theta_p^T(k)F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\nu_p^o(k+1) + y_u(k+1). \tag{D.9}$$

Substituting (D.9) into an incremented version of (D.8) yields

$$\begin{aligned}\nu_p^o(k+1) &= [1+\theta_p^T(k)F_p(k)\theta_p(k)][(a+c)e_p(k)+\theta_p^T(k)\Delta p_p(k)+y_u(k+1)] \\ &= \theta_p^T(k)F_p(k)\theta_p(k)\nu_p^o(k+1)\end{aligned} \tag{D.10}$$

or

$$\nu_p^o(k+1) = (a+c)e_p(k) + \theta_p^T(k)\Delta p_p(k) + y_u(k+1). \tag{D.11}$$

Substitute (D.4), (D.5), and (D.11) into (D.2) yielding

$$\begin{aligned}e_p(k+1) &= Ae_p(k)+b\theta_p^T(k)\Delta p_p(k)+by_u(k+1) \\ &\quad - \frac{b\theta_p^T(k)F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}[(a+c)e_p(k)+\theta_p^T(k)\Delta p_p(k)+y_u(k+1)]\end{aligned} \tag{D.12}$$

or

$$\begin{aligned}e_p(k+1) &= \left[A-\frac{\theta_p^T(k)F_p(k)\theta_p(k)b(a+c)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\right]e_p(k)+b\left[I-\frac{\theta_p^T(k)F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\right]\theta_p^T(k)\Delta p_p(k) \\ &\quad + b\left[I-\frac{\theta_p^T(k)F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)}\right]y_u(k+1).\end{aligned} \tag{D.13}$$

Similarly substitute (D.11) into (D.5), then

$$\Delta p_p(k+1) = \Delta p_p(k) - \frac{F_p(k)\theta_p(k)}{1+\theta_p^T(k)F_p(k)\theta_p(k)} [(a+c)e_p(k) + \theta_p^T(k)\Delta p_p(k) + y_u(k+1)]. \tag{D.14}$$

After regrouping (D.14), (D.13) and (D.14) can be compactly written as in (3.26) to (3.28).

APPENDIX E. SERIES-PARALLEL IDENTIFIER ERROR SYSTEM

This appendix derives (3.53) to (3.55) from (3.12) and (3.44) to (3.50). Subtracting (3.44) from (3.12) yields

$$y(k) - y_s(k) = \sum_{i=1}^{n}[a_i - \hat{a}_i(k)]y(k-i) + \sum_{i=1}^{n}[b_i - \hat{b}_i(k)]u(k-i) + y_u(k) \tag{E.1}$$

or using (3.46), (3.47), (3.52),

$$y(k+1) - y_s(k+1) = \theta^T(k)\Delta p_s(k+1) + y_u(k+1) \tag{E.2}$$

where

$$\Delta p_s(k) = [a_1 \ \dots \ a_n \ \ b_1 \ \dots \ b_n]^T - \hat{p}_s(k). \tag{E.3}$$

From (3.45), (3.49), (3.50), and (E.3)

$$\Delta p_s(k+1) = \Delta p_s(k) - \frac{F_s(k)\theta(k)[\theta^T(k)\Delta p_s(k) + y_u(k+1)]}{1 + \theta^T(k)F_s(k)\theta(k)} . \tag{E.4}$$

Regrouping (E.4) and substituting it into (E.2) verifies the parametrization of (3.53) to (3.55).

REFERENCES

1. P. C. Parks, "Liapunov Redesign of Model Reference Adaptive Control Systems," IEEE Trans. on Automatic Control, Vol. AC-11, pp. 362-367, July 1966.

2. K. S. Narendra, "Stable Identification Schemes," System Identification: Advances and Case Studies, Academic Press, New York, 1976.

3. G. Kreisselmeier, "The Generation of Adaptive Law Structures for Globally Convergent Adaptive Observers," IEEE Trans. on Automatic Control, Vol. AC-24, No. 3, pp. 510-512, June 1979.

4. R. L. Caroll and D. P. Lindorff, "An Adaptive Observer for Single-Input, Single-Output Linear Systems," IEEE Trans. on Automatic Control, Vol. AC-18, No. 5, pp. 428-435, October 1973.

5. R. V. Monopoli, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. on Automatic Control, Vol. AC-19, pp. 474-484, October 1974.

6. I. D. Landau, Adaptive Control: The Model Reference Approach, Marcel Dekker, New York, 1979.

7. K. S. Narendra and L. S. Valavani, "Stable Adaptive Controller Design-Direct Control," IEEE Trans. on Automatic Control, Vol. AC-23, No. 4, pp. 570-583, August 1978.

8. K. S. Narendra, Y. H. Lin, and L. S. Valavani, "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE Trans. on Automatic Control, Vol. AC-25, No. 3, pp. 440-448, June 1980.

9. A. Feuer and S. Morse, "Adaptive Control of Single-Input Single-Output Linear Systems," IEEE Trans. on Automatic Control, Vol. AC-23, pp. 557-570, August 1978.

10. G. Kreisselmeier, "Adaptive Control via Adaptive Observation and Asymptotic Feedback Matrix Synthesis," IEEE Trans. on Automatic Control, Vol. AC-25, August 1980.

11. G. C. Goodwin, P. S. Ramadge, and P. E. Caines, "Discrete Time Multivariable Adaptive Control," IEEE Trans. on Automatic Control, Vol. AC-25, June 1980.

12. K. J. Åström, U. Borisson, L. Ljung, and B. Wittenmark, "Theory and Applications of Self-Tuning Regulators," Automatica, Vol. 13, No. 5, pp. 457-476, September 1977.

13. C. R. Johnson, Jr., "Input Matching, Error Augmentation, Self-Tuning, and Output Error Identification: Algorithmic Similarities in Discrete Adaptive Model Following," IEEE Trans. on Automatic Control, Vol. AC-25, pp. 697-703, August 1980.

14. A. S. Morse, "Global Stability of Parameter Adaptive Control Systems," IEEE Trans. on Automatic Control, Vol. AC-25, pp. 433-439, June 1980.

15. G. Kreisselmeier, "On Adaptive State Regulation," IEEE Trans. on Automatic Control, Vol. AC-27, February 1982.

16. H. Elliott and W. A. Wolovich, "Parameter Adaptive Identification and Control," IEEE Trans. on Automatic Control, Vol. AC-24, pp. 592-599, August 1979.

17. I. D. Landau, "Adaptive Controllers with Explicit and Implicit Reference Models and Stochastic Self-Tuning Regulators - Equivalence and Duality Aspects," Proc. of 17th IEEE Conf. on Decision and Control, San Diego, CA, 1979.

18. K. S. Narendra and L. S. Valavani, "A Comparison of Lyapunov and Hyperstability Approaches to Adaptive Control of Continuous Systems," IEEE Trans. on Automatic Control, Vol. AC-25, pp. 243-247, April 1980.

19. B. Egardt, "Unification of Some Discrete-Time Adaptive Control Schemes," IEEE Trans. on Automatic Control, Vol. AC-25, pp. 693-697, August 1980.

20. P. A. Ioannou and P. V. Kokotovic, "Error Bounds for Model-Plant Mismatch in Identifiers and Adaptive Observers," Coordinated Science Lab. Rept. R-897, Univ. of Illinois, October 1980.

21. B. D. O. Anderson and C. R. Johnson, Jr., "On Reduced Order Adaptive Output Error Identification and Adaptive IIR Filtering," IEEE Trans. on Automatic Control, Vol. AC-27, August 1982, to appear.

22. C. R. Johnson, Jr. and M. S. Balas, "Reduced-Order Adaptive Controller Studies," Proc. of Joint Automatic Control Conference, San Francisco, CA, August 1980.

23. C. E. Rohrs, L. Valavani, and M. Athans, "Convergence Studies of Adaptive Control Algorithms, Part I: Analysis," Proc. 19th IEEE Conf. on Decision and Control, Albuquerque, NM, December 1980.

24. C. E. Rohrs, L. Valavani, M. Athans, and G. Stein, "Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms," Proc. 20th IEEE Conf. on Decision and Control, San Diego, CA, December 1981.

25. P. V. Kokotovic, R. E. O'Malley, Jr., and P. Sannuti, "Singular Perturbations and Order Reduction in Control Theory - An Overview," Automatica, Vol. 12, pp. 123-132, 1976.

26. P. V. Kokotovic, "A Riccati Equation for Block Diagonalization of Ill-Conditioned Systems," IEEE Trans. on Automatic Control, pp. 812-814, December 1975.

27. A. P. Morgan and K. S. Narendra, "On the Uniform Asymptotic Stability of Certain Linear Nonautonomous Differential Equations," SIAM J. Control and Optimization, Vol. 15, No. 1, pp. 5-24, January 1977.

28. B. D. O. Anderson, "Exponential Stability of Linear Equations Arising in Adaptive Identification," IEEE Trans. on Automatic Control, Vol. AC-22, pp. 83-88, February 1977.

29. J. C. Yuan and W. M. Wonham, "Probing Signals for Model Reference Identification," IEEE Trans. on Automatic Control, pp. 530-538, August 1977.

30. P. V. Kokotovic and A. H. Haddad, "Controllability and Time-Optimal Control of Systems with Slow and Fast Modes," IEEE Trans. on Automatic Control, Vol. AC-20, pp. 111-113, February 1975.

31. J. H. Chow, "Preservation of Controllability in Linear Time-Invariant Perturbed Systems," Int. J. Control, Vol. 25, No. 5, pp. 697-704, 1975.

32. P. Kudva and K. S. Narendra, "Synthesis of an Adaptive Observer Using Lyapunov's Direct Method," Int. J. of Control, Vol. 18, pp. 1201-1210, December 1973.

33. K. S. Narendra and P. Kudva, "Stable Adaptive Schemes for System Identification and Control, Part II," IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC-4, No. 6, pp. 552-560, November 1974.

34. G. Kreisselmeier, "Adaptive Observers with Exponential Rate of Convergence," IEEE Trans. on Automatic Control, Vol. AC-19, pp. 117-119, April 1974.

35. R. G. Phillips, "Reduced Order Modeling and Control of Two-Time-Scale Discrete Systems," Int. J. Control, Vol. 31, No. 4, pp. 765-780,

36. B. D. O. Anderson and C. R. Johnson, Jr., "Exponential Convergence of Adaptive Identification and Control Algorithms," Univ. of Newcastle, Dept. of Etectrical Engineering Technical Rept., April 1980.

37. C. A. Desoer and M. Vidyasagar, Feedback Systems: Input-Output Properties, Academic Press, 1975.

38. J. M. Mendel, Discrete Techniques of Parameter Estimation: The Equation Formulation, Marcel Dekker, Inc., New York, 1973.

39. B. B. Peterson and K. S. Narendra, "Bounded Error Adaptive Control," Part I, S&IS Rept. No. 8005, Dept. of Engineering and Applied Science, Yale University, New Haven, CT, December 1980.

40. H. K. Khalil, "On the Robustness of Output Feedback Control Methods to Modeling Errors," IEEE Trans. on Automatic Control, Vol. AC-26, April 1981.

41. P. A. Ioannou, "Robustness of Absolute Stability," Int. J. Control, Vol. 34, No. 5, pp. 1027-1033,

42. M. Aoki, "On Feedback Stabilizability of Decentralized Dynamic Systems," Automatica, Vol. 8, pp. 163-173, March 1972.

43. S. H. Wang and E. J. Davison, "On the Stabilization of Decentralized Control Systems," IEEE Trans. on Automatic Control, Vol. AC-18, No. 5, pp. 473-478, October 1973.

44. M. Darwish, H. M. Soliman, and J. Fantin, "Decentralized Stabilization of Large-Scale Dynamical Systems," IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC-9, No. 11, pp. 717-720, November 1979.

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